20090401

Substitution (Integrating the Chain rule)

$$\frac{d}{dx} F(u(x)) = F'(u(x))\, u'(x)\, dx$$

$$\int F'(u)\, u'\, dx = \int F'(u)\, du = F(u) + C$$

$u = u(x)$

if integrand "$g(u)\, u'$"

Example. (Memo) $\int 2e^{2x}\, dx \underset{Re}{=} e^{2x} + C$

$u = 2x$
$du = 2dx$; gather $\int e^u\, du = e^u + C$

but $u = 2x$ $\int e^u\, du = e^{2x} + C$

More meaningful $\int e^{2t-1}\, dt$ Choose a variable

(several approaches) $u = 2t - 1$ → forced $du = \ldots d$
$du = 2\, dt$ Try — plug in

BLOCKED? $dt = \frac{1}{2}\, 2dt$

$$\int e^u \tfrac{1}{2}\, du = \tfrac{1}{2}\int e^u\, du = \tfrac{1}{2} e^u + C$$

$$= \tfrac{1}{2} e^{2t-1} dt$$

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IVP (Initial Value Problems

Ex 5.1.5 Marginal cost

$$\frac{dC}{dq} = 3q^2 - 60q + 400 \leftarrow \text{DE}$$

$$C(2) = 900 \qquad \leftarrow \text{Initial Value IV}$$

Find $C(5)$; Find $C(x)$

Prob $\boxed{32}$ $\frac{dy}{dx} = e^{-x}$, $y = 3$ when $x = 0$

$$y = \int e^{-x}\,dx = -e^{-x} \underbrace{+\ C}_{\text{adjoin}}$$

$$y(0) = y(x)\Big|_{x=0} = 3 = -e^{-0} + C$$

$$\therefore C = 4$$

$$\boxed{y = -e^{-x} + 4}$$

#45 $MC = \frac{dC}{dq} = 3q^2 - 24q + 48$ $\qquad C\Big|_{x=20} = 30'000$

(root ~~$+B \pm \sqrt{\ }$~~

$$\frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{+24 \pm \sqrt{(24)^2 - 196}}{2}$$ pos

(Not reasonable")

$$C(q) = \int \frac{dC}{dq}\,dq = q^3 - 12q^2 + 48q + C$$

$$C(20) = 30000 = (20)^3 - 12(20)^2 + 48(20) + C$$

$$C = \ldots$$

On some integrals, get lucky

$\boxed{\text{Prob 5}}$ $\int \sqrt{4x-1}\, dx$ (know $\int \sqrt{u}\, du$

$u = 4x-1$
$du = 4dx = \frac{1}{4}$;

$$\int \sqrt{u}\, \frac{1}{4} du = \frac{1}{4} \int \sqrt{u}\, du = \frac{1}{4} \frac{u^{3/2}}{3/2} = \frac{1}{6} (4x-1)^{3/2} + C$$

IVP 45 $x'(t) = -2(3t+1)^{1/2}$ $\boxed{x(0)=4}$

$$x(t) = \int -2(3t+1)^{1/2} dt$$

-2 for
$u=(3t+1)$
$du = 3dt =$
$dt = \frac{1}{3}a$

BOOKKEEPING

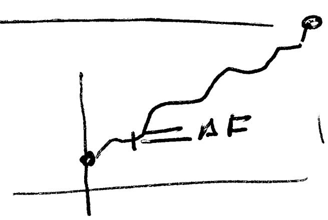
$$= -2 \int u^{1/2} \cdot \frac{1}{3} du$$

$$= -\frac{2}{3} \cdot \frac{u^{3/2}}{3/2} = -\frac{4}{9} (3t+1)^{3/2} + C$$

Find C: $x(0) = 4 = -\frac{4}{9}(0+1)^{3/2} + C$ $C = 4\frac{4}{9}$

Definite Integral

$F(b) - F(a)$



$$= \sum \Delta F \approx \sum F'(x)\, \Delta x \text{ Stuff by area}$$

$$\to \text{"}\int_a^b F'(x)\, dx.$$