Partial Derivatives

~~fcn~~ $z = f(x, y)$

$\frac{\partial z}{\partial x} \equiv$ process {freeze y, consider as fn of x, take derivative}

$\longrightarrow \frac{\partial z}{\partial x} = $ fn of (x, y) (both "usually

$\frac{\partial z}{\partial y} =$

∂ Ex $f(x,y) = \frac{x}{y}$ $[y \neq 0]$

$\frac{\partial f}{\partial x} = \frac{1}{y} \cdot 1$ $\frac{\partial f}{\partial y} = x \left(\frac{d}{dy} \frac{1}{y}\right) = -\frac{x}{y^2}$

∂ Ex 7.2 prob 3 $z = xe^{xy}$

$\frac{\partial z}{\partial x} = 1 \cdot e^{xy} + x \frac{\partial}{\partial x} e^{xy}$

$= e^{xy} + x \cdot y e^{xy} = e^{xy}(1 + xy)$

$\frac{\partial z}{\partial y} = \underset{\uparrow \text{const.}}{x} \; x e^{xy} = x^2 e^{xy}$

Prob Where are both $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y} = 0$?

$1 + xy = 0$, $x^2 = 0$

$x = 0$ <u>never</u>!

Revenue from Two Products (33)

x – production to domestic $\quad x+y = \text{const}?$

y – production to foreign

Domestic Price $= 60 - \frac{x}{5} + \frac{y}{20}$

Export Price $= 50 - \frac{y}{10} + \frac{x}{20}$

$R(x,y) = x \text{ domestic} + y \text{ export}$

$= x\left(60 - \frac{x}{5} + \frac{y}{20}\right) + y\left(50 - \frac{y}{10} + \frac{x}{20}\right)$

$= \ldots$

$dR = \frac{\partial R}{\partial x}dx + \frac{\partial R}{\partial y}dy$

$= \left(-\frac{2}{5}x + 60 + \frac{1}{20}y\right)dx + \left(\frac{x}{20}\right) - \frac{2y}{10} + \frac{y}{20}\Big)$

Approximation of Change (Linear "Tangent Line Approx

① $\quad y = f(x)$

Change x by Δx (small

$\Delta y \approx \frac{df}{dx}\Delta x$

$dy \equiv \frac{dy}{dx}dx$ Meaning:

Change of x by Δx (small) gives $\Delta y \approx \frac{\partial f}{\partial x}\Delta x$,

(better (error $\ll |\Delta x|$

② 2 variables Look at domain x, y

$F(x,y)$

Δx

(x,y) Δy

Change from (x,y) to $(x+\Delta x, y+\Delta y)$

change here Δx, Δy

$= $ change here Δx, change here $\approx \underbrace{\frac{\partial f}{\partial x}}_{\text{rates}} \Delta x + \underbrace{\frac{\partial f}{\partial y}}_{\text{rate}} \Delta y$

The differential of a function is $d\cancel{f}z = \frac{\partial z}{\partial x} dx + \frac{\partial f}{\partial y} dy$

$\boxed{\text{Ex 7.2.5}}$

Production $Q(K,L) = 50K^{0.4}L^{0.6}$

(get your calculator out)

K = capital $\quad L$ = worker hours.

"Marginal Productivity of Kapital" $Q_K \equiv \frac{\partial Q}{\partial K}$

($\approx$ how much Q changes if amount of Kapital is added) 1 ~~unit~~ small unit K

"Marginal Prod of Labor" - - - $Q_L \equiv \frac{\partial Q}{\partial L}$

Suppose $K = 750$ (thousand)

$L = 991$.

$Q_K = \frac{\partial Q}{\partial K} = 50\left[0.4K^{-0.6} L^{0.6}\right] = 23.63$

$Q_L = \frac{\partial Q}{\partial L} = 50\left[0.6 K^{0.4} L^{-0.4}\right]$ 26.84

750 991

when $\boxed{K = 750 \,//\, L = 991}$

23.63 per \$1000 Kap

26.84 per worker-hr

Considering cost better strategy

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Above are called $\frac{\partial f}{\partial x}$ $\left(\frac{\partial Q}{\partial K}\right.$, etc

are first order partial der..

"Higher Order" (in practice 2)

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \qquad \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\right) \qquad \frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\right.\partial z$$

Fortunately the "mixed partials" are the same

<u>Chain Rule</u>

$$\frac{d}{dt} f(\cancel{x,t})(x(t), y(t)) = \left.\frac{\partial f}{\partial x}\right|_{\text{eval at}} \frac{dx}{dt} + \left.\frac{\partial f}{\partial y}\right|_{\text{eval at}} \frac{dy}{dt}$$