samplefinal2008.mw
Maple 10 Worksheet for samplefinal2008 Math 165 - Calculus for Business.
Sample Final 2008 (Problems 1-62 are from samplefinal2007)
First load plots and student:

Definite integrals are written and evaluated using Maple commands Int and int.
int(f(x), x = a .. b) is the equivalent (almost) of your TI calculator calculator fnInt(f(x),x,a,b).
An alternate method to evaluate the definite integral is value(Int).

$$InertDefiniteIntegral := \int_0^{1.0} e^{2x-1}\, dx$$

$$EvaluatedDefiniteIntegral := 1.175201194$$

$$ValueInt := 1.175201194 \qquad (1)$$

Sample Final 2008 (Problems 1-62 are from samplefinal2007)

2008 good 1. Suppose z = f(x, y), f_x(a,b) = f_y(a,b)=0 f_xx=16,f_yy=16 ,f_xy=16
and (,) 4 xy f a b = . Then (a, b) is
A) the test is inconclusive
B) a saddle point
C) a relative minimum
D) a relative maximum
D = fxx*fyy -fxy*fxy
Discriminant Test Fails

$$DISC := f\_xx*f\_yy - f\_xy*f\_xy$$

$$D := 0 \qquad (2)$$

2008 good 2.. The only grocery store in a small rural community carries two brands of frozen apple juice, a local brand that it obtains at the cost of 18 cents per can and a well-known national brand that it obtains at the cost of 60 cents per can. The grocer estimates that if the local brand is sold for x cents per can and the national brand for y cents per can, approximately 70 – 5x + 4y cans of the local brand and 80 + 6x-7y cans of the national brand will be sold each day. How should the grocer price each brand to maximize the profit from the sale of the juice?
A) local brand (x) at 74 cents, national brand (y) at 55 cents
B) local brand (x) at 32 cents, national brand (y) at 114 cents
C) local brand (x) at 39 cents, national brand (y) at 57 cents
D) local brand (x) at 37 cents, national brand (y) at 57 cents
Profit on each brand = (price - cost) * (demand/quantity)

$$P\_local(x, y) := (x-18)\,(70-5x+4y)$$

$$P\_national(x, y) := (y-60)\,(80+6x-7y)$$

$$Pr(x, y) := (x-18)\,(70-5x+4y) + (y-60)\,(80+6x-7y)$$

$$-200x - 5x^2 + 10xy - 6060 + 428y - 7y^2$$

$$Profit := \textbf{proc}(x, y)\ (x-18)*(70-5*x+4*y) + (y-60)*(80+6*x-7*y)\ \textbf{end proc}$$

$$Profit(x,y) := (x-18)\,(70-5x+4y) + (y-60)\,(80+6x-7y)$$

$$Simplified := -200x - 5x^2 + 10xy - 6060 + 428y - 7y^2 \qquad (3)$$

Now take the first partial derivatives and solve the two simultaneous equations to find the critical point. Then the discriminant test will show a local.global MAX

$$Pr\_x(x, y) := -200 - 10\,x + 10\,y$$

$$Pr\_y(x, y) := 10\,x + 428 - 14\,y$$

$$Pr\_xx(x, y) := -10$$

$$Pr\_xy(x, y) := 10$$

$$Pr\_yy(x, y) := -14$$

$$Answer := \{y = 57, x = 37\}$$

$$critical\_point := [37, 57] \qquad (4)$$

2008 3. Compute f _yy for f (x, y) = e^(4xy) . A

$$f(x, y) := e^{4\,x\,y}$$

$$f\_y(x, y) := 4\,x\,e^{4\,x\,y}$$

$$f\_yy(x, y) := 16\,x^2\,e^{4\,x\,y} \qquad (5)$$

2008 4. . Find the second partial f_xy given f (x, y) = 5x e^(5xy) + y log(2x + 9y)
TYPESETTING error. Must reinterpret problem! () missing. log maens ln. Correct answer to problem with () is D

$$f(x, y) := 5\,x\,e^{8\,x\,y} + y\ln(2\,x + 9\,y)$$

$$f\_y(x, y) := 40\,x^2\,e^{8\,x\,y} + \ln(2\,x + 9\,y) + \frac{9\,y}{2\,x + 9\,y}$$

$$f\_x(x, y) := 5\,e^{8\,x\,y} + 40\,x\,y\,e^{8\,x\,y} + \frac{2\,y}{2\,x + 9\,y}$$

$$f\_xy(x, y) := 80\,x\,e^{8\,x\,y} + 320\,x^2\,y\,e^{8\,x\,y} + \frac{2}{2\,x + 9\,y} - \frac{18\,y}{(2\,x + 9\,y)^2} \qquad (6)$$

Page 2.
2008 5. Find the second partials (including the mixed partials) of f (x, y) = 7 x^5 y^6 + 5 x y

$$f(x, y) := 7\,x^5\,y^6 + 5\,x\,y$$

$$f\_y(x, y) := 42\,x^5\,y^5 + 5\,x$$

$$f\_xy(x, y) := 210\,x^4\,y^5 + 5$$

$$f\_xx(x, y) := 140\,x^3\,y^6$$

$$f\_yy(x, y) := 210\,x^5\,y^4 \qquad (7)$$

2008 6.. Compute f(ln2, ln7) if f (x, y) = e^(2x+ y) .

$$ans\_7 := e^{2\ln(2) + \ln(7)}$$

$$or := 28 \qquad (8)$$

2007 7.. Let f (x) =10*x^9 - 180 * ln(x) , for x > 0. Find the minimum value of f for x > 0.
There is a local minimum by the second derivative test; as x -> 0+ , f(x) -> + infty.
As x -> + infty, f(x) -> +infty

$$f(x) := 10\,x^9 - 180\ln(x)$$

$$f\_prime(x) := 90\,x^8 - \frac{180}{x}$$

$$crit\_no := 2^{1/9}$$

$$Answer := 20 - 20\ln(2) \tag{9}$$

2007 8.. Suppose your family owns a rare book whose value t years from now will be
V(t) = 9*exp(sqrt(7.*t)) dollars. If the prevailing interest rate remains constant at 6% per
year compounded continuously, when will it be most advantageous for your
family to sell the book and invest the proceeds?
See Example 4.4.3 page 336.
Method 1: Maximize the Present Value

$$Present(t) := 9\,e^{-0.06\,t}\,e^{2.645751311\sqrt{t}}$$

$$Present\_prime(t) := -0.54\,e^{-0.06\,t}\,e^{2.645751311\sqrt{t}} + \frac{11.90588090\,e^{-0.06\,t}\,e^{2.645751311\sqrt{t}}}{\sqrt{t}}$$

$$Answer := 486.1111111 \tag{10}$$

Method 2.
We actually want to know when the relative rate of return (or percentage rate of return) of the two methods of investment are =.

$$Book(t) := 9\,e^{\sqrt{7}\sqrt{t}}$$

$$P(t) := e^{0.06\,t}$$

$$Book\_Rate(t) := \frac{1}{2}\,\frac{\sqrt{7}}{\sqrt{t}}$$

$$P\_Rate(t) := 0.06$$

$$Change\_Date := 486.1111111 \tag{11}$$

2008 8 different data.. Suppose your family owns a rare book whose value t years from now will be
V(t) = 4e^sqrt(5t) dollars. If the prevailing interest rate remains constant at 6% per
year compounded continuously, when will it be most advantageous for your
family to sell the book and invest the proceeds?
A) 416.67 years
B) 486.11 years
C) 555.56 years
D) 625 years=
See Example 4.4.3 page 336.
Method 1: Maximize the Present Value

$$Present(t) := 4\,e^{-0.06\,t}\,e^{2.236067977\sqrt{t}}$$

$$Present\_prime(t) := -0.24\,e^{-0.06\,t}\,e^{2.236067977\sqrt{t}} + \frac{4.472135952\,e^{-0.06\,t}\,e^{2.236067977\sqrt{t}}}{\sqrt{t}}$$

$$Change\_date := 347.2222218 \tag{12}$$

Method 2.
We actually wnat to know when the rate of returns of the two methods of investment are =.

$$Book(t) := 4\, e^{2.236067977\sqrt{t}}$$

$$P(t) := e^{0.06\, t}$$

$$Book\_Rate(t) := \frac{1.118033988}{\sqrt{t}}$$

$$P\_Rate(t) := 0.06$$

$$Change\_Date := 347.2222218 \quad (13)$$

2008 9. How quickly will money triple if it is invested at 7% interest compounded continuously?

A) 15.69 years
B) 15.71 years
C) 15.73 years
D) 15.75 years

Note that the answer does not depend in the initial balance B.

$$eqn := B\_0\, e^{0.07\, T} = 3\, B\_0$$

$$ans\_9 := 15.69446127 \quad (14)$$

2008 10.
. Find df/dx where .f(x):=ln(x^6)
Two ways - ln(x^6) or 6*ln(x)

$$f(x) := \ln(x^6)$$

$$ans\_10 := \frac{6}{x} \quad (15)$$

2008 11.. The equation of the tangent line to y = 8*ln(x^3) at x = e is

$$f11(x) := 8\ln(x^3)$$

$$f11\_prime(x) := \frac{24}{x}$$

$$slope := \frac{24}{e}$$

$$tan\_eqn(x) := 24 + \frac{24\,(x - e)}{e}$$

$$Answer\_11{:}\ y := 24\, x\, e^{-1}$$

$$Tangent\ Line\ at\ x = 2\ is\ y := \frac{24\, x}{e} \quad (16)$$

2008 12.. Find the derivative of
Best to use ln(((ln(x^2)))^3) = 3 ln (2 ln(x)) = 3 ln 2 + 3 ln(ln(x))

$$f(x) := \ln\left(\ln(x^2)^3\right)$$

$$f\_prime(x) := \frac{6}{\ln(x^2)\, x}$$

(17)

$$Answer\_12 := \frac{6}{\ln(x^2)\, x} \tag{17}$$

2008 13. The consumer demand for a certain commodity is 7000 exp(-.052p units per month when the market price is p dollars per unit. Express consumers'total monthly expenditure for the commodity as a function of p and determine the market price that will result in the greatest consumer expenditure.
Here "total monthly expenditure" means the usual revenue.

$$D\_13(p) := 7000\, e^{-0.52\,p}$$

$$R\_13(p) := 7000\, p\, e^{-0.52\,p}$$

$$ddp\_R\_13(p) := 7000\, e^{-0.52\,p} - 3640.00\, p\, e^{-0.52\,p}$$

$$crit\_13 := 1.923076923 \tag{18}$$

2008 14. Use logarithmic differentiation to find f'(x); f(x) = ((6*x + 9)/(4 + 4*x))^(1/4)
(By hand is easier!)
Good practice! .

$$f(x) := \left(\frac{6\,x+9}{4+4\,x}\right)^{1/4}$$

$$log\_f(x) := \frac{1}{4}\ln\left(\frac{6\,x+9}{4+4\,x}\right)$$

$$simplified := \frac{1}{2}\, 3^{1/4}\sqrt{2}\left(\frac{2\,x+3}{1+x}\right)^{1/4}$$

$$d\_dx\_log\_f(x) := \frac{1}{4}\,\frac{\left(\frac{6}{4+4\,x} - \frac{4\,(6\,x+9)}{(4+4\,x)^2}\right)(4+4\,x)}{6\,x+9}$$

$$or := -\frac{1}{4\,(1+x)} + \frac{1}{2\,(2\,x+3)}$$

$$Answer := \left(\frac{6\,x+9}{4+4\,x}\right)^{1/4}\left(-\frac{1}{4\,(1+x)} + \frac{1}{2\,(2\,x+3)}\right) \tag{19}$$

2008 15.. Solve for x: .(Easier by hand!)

$$eqn\_15 := 5\ln(x) - \frac{1}{7}\ln(x^5) = 30$$

$$eqn := \frac{30}{7}\ln(x) = 30$$

$$divide\_by\_30 := \frac{1}{7}\ln(x) = 1$$

$$multipy\_by\_7 := \ln(x) = 7$$

$$exponentiate := e^{\ln(x)} = e^7$$

$$answer := e^7 \tag{20}$$

2008 16. A radioactive substance decays exponentially. If 600 grams were present

initially and 200 grams are present 100 years later, how many grams will be present after 400 years?
A) 7.41 grams
B) 0 grams
C) 6.16 grams
D) 4.91

$$R(t) := 600 \left(\frac{1}{3}\right)^{\frac{1}{100}t}$$

$$fraction\ answer := \frac{200}{27}$$

$$Answer\_16 := 7.407407407 \quad (21)$$

2008 17.. Solve for x: log_2(x) is the * s.t. 2^* = x.

$$eqn\_17 := \frac{\ln(x-1)}{\ln(2)} = 5$$

$$2^{\frac{\ln(x-1)}{\ln(2)}} = 32$$

$$x-1 = 32$$

$$Answer := 33$$

$$solution := 33 \quad (22)$$

2008 18.
If $1500 is invested at 10 percent compounded continuously, what is the balance after 12 years?

$$P(t) := 1500\, e^{0.10\, t}$$

$$4980.175384 \quad (23)$$

2008 19. Find all the critical points of the function-2*x^4 + 4*x^2 +1

$$f(x) := -2\, x^4 + 4\, x^2 + 1$$

$$f\_prime(x) := -8\, x^3 + 8\, x$$

$$crit\_no := 0, 1, -1 \quad (24)$$

2008 20.. Find the intervals of increase and decrease for .f(x):=(10*x - 3)/(-x + 10). Notice x =10 is a vertical asymptote.

$$f(x) := \frac{10\, x-3}{-x+10}$$

$$f\_prime(x) := \frac{10}{-x+10} + \frac{10\, x-3}{(-x+10)^2}$$

$$simplified := \frac{97}{(x-10)^2}$$

$$increase\ (f'>0) := RealRange(-\infty, Open(10)), RealRange(Open(10), \infty) \quad (25)$$

2008 21 Find the intervals of increase and decrease for the function f(x):=x^2 + 3*x -7

$$f(x) := x^2 + 3x - 7$$

$$f\_prime(x) := 2x + 3$$

$$crit\_no := -\frac{3}{2}$$

$$increase\ (f'>0) := RealRange\left(Open\left(-\frac{3}{2}\right),\ \infty\right)$$

$$decrease(f' < 0) := RealRange\left(-\infty,\ Open\left(-\frac{3}{2}\right)\right) \quad (26)$$

200822. Determine where the graph of x^3 - 3*x^2 - 9 *x +1 is concave down.

A) x > 1
B) x < 1
C) x > -1
D) x < -1

$$f := x \to x^3 - 3x^2 - 9x + 1$$

$$f\_prime(x) := 3x^2 - 6x - 9$$

$$f\_prime\_prime(x) := 6x - 6$$

$$possible\_inflection := 1$$

$$concave\_up\ (f'' > 0) := RealRange(Open(1),\ \infty)$$

$$concave\_down\ (f'' > 0) := RealRange(-\infty,\ Open(1)) \quad (27)$$

2008 23. A 5-year projection of population trends suggests that t years from now, the population of a certain community will be
-t^3 + 9 * t^2 +120*t +55 thousand.

1) At what time during the 5-year period will the population be growing most rapidly?
2) At what time during the 5-year period will the population be growing least rapidly?
3) At what time is the rate of population growth changing most rapidly?

A) t = 5 years; t = 0 years; t = 0 years
B) t = 0 years; t = 0 years; t = 2 years
C) t = 2 years; t = 1 year; t = 3 years
D) t = 2 years; t = 0 years; t = 2 years

$$P := t \to -t^3 + 9t^2 + 120t + 55$$

$$P\_prime(t) := -3t^2 + 18t + 120$$

$$-3(t+4)(t-10)$$

$$P\_prime\_prime(t) := -6t + 18$$

$$crit\_no := 3$$

$$rate\ changing\ most\ rapidly := 18,\ \{[\{t=0\},\ 18]\} \quad (28)$$

P' is INCreasing on [0,3] and achieves MAX at t=3., MIN at t=0. |P"(t)| is MAX at t = 0.
Answer A

2008 24.. Find the absolute maximum of the function x^5 - x^4 on the interval – 1 <= x

<= 1 .

A) 0
B) 1
C) -1
D) -2

Check the critical numbers and the endpoints.

Careful: MAX value of f(x) is requested; graph shows f(0) = f(1) = 0 and for 0 < x < 1, f(x) <0.

$$f\_prime(x) := 5\,x^4 - 4\,x^3$$

$$crit\_no := \frac{4}{5},\ 0,\ 0,\ 0$$

$$-\frac{256}{3125}$$

$$endpoints\_and\_critical\_numbers := [-1, 1, 0, 0.8]$$

$$Points\_on\_graph := [[-1, -2], [1, 0], [0, 0], [0.8, -0.08192]]$$

$$answer := \textit{max value is f(0) =f(1) = 0} \tag{29}$$

2008 25.. An apartment complex has 250 units. When the monthly rent for each unit is $400, all units are occupied. Experience indicates that for each $12 per month increase in rent, 4 units will become vacant. Each rented apartment costs the owner of the complex $40 per month to maintain. What monthly rent should be charges to maximize profit?

Formula for demand is q = 250 (all) - (4 per $12)* (increase in rent from $400).
Profit per rented unit is p - 40).

$$q(p) := \frac{1150}{3} - \frac{1}{3}\,p$$

$$Profit(p) := (p-40)\left(\frac{1150}{3} - \frac{1}{3}\,p\right)$$

$$-\frac{1}{3}\,(p-40)\,(-1150+p)$$

$$-\frac{1}{3}\,(p-40)\,(-1150+p)$$

$$Profit\_prime(p) := \frac{1190}{3} - \frac{2}{3}\,p$$

$$\textit{Maximum Profit at Rent} := 595 \tag{30}$$

2008 26. A commuter's train carries 600 passengers each day from a suburb to a city. It now costs $1 per person to ride the train. A study shows that 50 additional people will ride the train for each 5 cent reduction in fare. What fare should be charged in order to maximize total revenue?

A) 78 cents
B) 79 cents
C) 80 cents
D) 85 cents

Simlar to previous problem. q = number riders, p = fare

$$q(p) := 1600.000000 - 1000.000000\,p$$

$$R(p) := p\ (1600.000000 - 1000.000000\ p)$$

$$R\_prime(p) := 1600.000000 - 2000.000000\ p$$

$$Maximum\ Revenue\ at\ fare := 0.8000000000 \quad (31)$$

2008. 27.
Find the elasticity n of the demand function . Recall price elasticity of demand is E(p)= (p/q)(dq/dp).

$$D\_27(p) := \frac{3}{1 + 2\,p^2}$$

$$diff\_d\_27 := -\frac{12\,p}{(1 + 2\,p^2)^2}$$

$$n\_27(p) := -\frac{4\,p^2}{1 + 2\,p^2} \quad (32)$$

2008 28 A Florida citrus grower estimates that if 70 orange trees are planted, the average yield per tree will be 300 oranges. The average yield will decrease by 3 oranges per tree for each additional tree planted on the same acreage. How many trees should the grower plant to maximize the total yield?

A) 70 trees
B) 20 trees
C) 30 trees
D) 65 trees

AY = average yield
TY = total yield = q * AY

$$AY(q) := 510 - 3\,q$$

$$TY(q) := q\ (510 - 3\,q)$$

$$d\_dq\_TY(q) := 510 - 6\,q$$

$$crit\_no\_30 := 85 \quad (33)$$

2008 29.
The owner of a novelty store can obtain joy buzzers from the manufacturer for 50cents apiece. He estimates he can sell 70 buzzers when he charges $1.4 apiece for them and that he will be able to sell 11 more buzzers for every 10 cent decrease in price. What price should he charge in order to maximize profit?

$$q(p) := 224.0000000 - 110.0000000\ p$$

$$Profit(p) := (p - 0.50)\ (224.0000000 - 110.0000000\ p)$$

$$Profit\_prime(p) := 279.0000000 - 220.0000000\ p$$

$$1.268181818$$

$$ans\_31 := 1.268 \quad (34)$$

2008 30.
The derivative of f(t):= 1/(t^7) is

$$f(t) := \frac{1}{t^7}$$

(35)

$$f\_prime(x) := -\frac{7}{t^8} \tag{35}$$

2008 31.. The graph of 5x^2 + 9 has
A) a maximum at
B) a minimum at
C) a maximum at x = 0
D) a minimum at x = 0

$$f(x) := 5x^2 + 9$$

$$f\_prime(x) := 10x$$

$$crit\_no := 0$$

$$second\_der\_test := 10 \tag{36}$$

2008 32. Differentiate:

$$f(x) := \sqrt{x} + \frac{3}{\sqrt{x}}$$

$$f\_prime(x) := \frac{1}{2\sqrt{x}} - \frac{3}{2x^{3/2}} \tag{37}$$

2008 33. Find the second derivative of the given function and simplify your answer:

$$f(t) := \frac{7}{4t+7}$$

$$f'(t) := -\frac{28}{(4t+7)^2}$$

$$f''(t) := \frac{224}{(4t+7)^3} \tag{38}$$

2008 34. Evaluate the limit:

$$f(x) := \frac{x^4 + 2x^2 - 3}{x^5}$$

$$Answer\_34 := 0 \tag{39}$$

2008 35. Evaluate int((8*x)^(1/3),x)

$$ans\_35(x) := \frac{3}{4} x^{4/3} 8^{1/3} + C$$

$$Answer := \frac{3}{2} x^{4/3} + C \tag{40}$$

2008 36. NO CORRECT ANSWER.CHECK ANSWER An object is moving so that its speed after 5 + 2*t +6*t^2 minutes is meters per minute.
How far does the object travel between the end of minute 5 and the end of minute 6?
A) 490 meters

B) 3002 meters
C) 480 meters
D) 2832 meters
Here the interal is from t = 5 to t =6; minute 1: t from 0 to 1, minute 2: t from 1 to 2; etc.

Use fnInt(5 + 2*t +6*t^2,t,5,6)

$$\int_5^6 \left(5 + 2t + 6t^2\right) \mathrm{d}t$$

$$ans\_38 := 198 \tag{41}$$

2008 37. Evaluate

$$Integral := \int \left(2x^8 - 5x + 6\right) \mathrm{d}x$$

$$Answer\_37 := \frac{2}{9}x^9 - \frac{5}{2}x^2 + 6x + C \tag{42}$$

2008 38. NO CORRECT ANSWER In a certain section of the country, the price of chicken is currently $2 per kilogram.
It is estimated that x weeks from now the price will be increasing at a rate of 2sqrt(x +1) **cents** per kilogram, per week. How much will chicken cost 9 weeks from now?

A) $3.61
B) $0.62
C) $4.61
D) $2.62

$$Integral := 2.00 + \int_0^9 0.02\sqrt{t+1}\,\mathrm{d}t$$

$$ans\_38 := 2.408303688 \tag{43}$$

2007 39. Use the fundamental theorem of calculus to find the area of the region under the line y = 2x + 1 above the interval 1 □ x □ 8 .
A) 74
B) 72
C) 70
D) 68
fnInt(8*x + 5,x,5,8) OK

$$Area := \int_1^8 (8x + 5)\,\mathrm{d}x$$

$$FTC\ says := \left(4x^2 + 5x\right)\Big|_{x=8} - \left(\left(4x^2 + 5x\right)\Big|_{x=5}\right)$$

$$ans\_41 := 171 \tag{44}$$

2008 40.
40 NO CORRECT ANSWER CHECK ANSWER An animal population increases at the rate of 8 x^3 + 3 per year. What is the approximate increase in the animal population from the end of year 1 to the

end of year 4?
A) 4611
B) 4621
C) 23066
D) 5399

$$Increase := \int_{1}^{4} (8\,x^3 + 3.)\,dx$$

$$ans\_42 := 519. \qquad (45)$$

2008 41. Records indicate that t hours past midnight, the temperature at the local airport was
- 0.2* t^3 + k*t^2 + 5 degrees Celsius. If the average temperature between 11 A.M. and 1 P.M. is 50 degrees C, what is k?
A) 1.90
B) 1.95
C) 2.05
D) 2.72

$$f(t) := -0.2\,t^3 + k\,t^2 + 5$$

$$\int_{11}^{13} (-0.2\,t^3 + k\,t^2 + 5)\,dt$$

$$AVG := \frac{1}{2}\int_{11}^{13} (-0.2\,t^3 + k\,t^2 + 5)\,dt$$

$$avg := -343.0000000 + 144.3333334\,k$$

$$ans\_43\ k := 2.722863740 \qquad (46)$$

2008 42. NO CORRECT ANSWER CHECK ANSWER Money is transferred continuously into an account at the constant rate of $1300 per year. The account earns interest at the annual rate of 7% compounded continuously. How much will be in the account at the end of 5 years?
Wrong Answer! -- None Correct
Formula: FV =fn Int(R e^(r(T-t)) ,t, 0,T)

2008 43. It is estimated that t days from now a farmers crop will be increasing at the rate of
0.3*t^2 + 0.6*t + 1
bushels per day. By how much will the value of the crop increase
during the next 2 days if the market price remains fixed at $5
It is estimated that t days from now a farmers crop will be increasing at the rate
of 0.3t 2 + 0.6t +1 bushels per day. By how much will the value of the crop
increase during the next 2 days if the market price remains fixed at $4 per
bushel?

$$FV := \int_{0}^{5} 1300\,e^{0.35 - 0.07\,t}\,dt$$

$$ans\_44 := 7782.683045$$

$$INCREASE := 4 \int_0^2 (0.3\, t^2 + 0.6\, t + 1)\, dt$$

$$ans\_45 := 16. \quad (47)$$

2008 44. Money is transferred continuously into an account at the constant rate of $1000 per year. Assume the account earns interest at the annual rate of 7% compounded continuously. Compute the future value of the income stream over a 14 year period.
Formula: FV =fn Int(R e^(r(T-t)) ,t, 0,T)

$$FV := \int_0^{14} 1000\, e^{0.98 - 0.07\, t}\, dt$$

$$ans\_46 := 23777.94631 \quad (48)$$

2008 45 Domain is {(x,y) | x^2 + y^2 > 16}

$$f(x, y) := \frac{x}{\sqrt{x^2 + y^2 - 16}}$$

```
Warning, solutions may have been lost
```

2008 46
46. The monthly demand for product A is D1 = 50 -10x + y , while D2 = 20 + 3x -5y is that for B. A sells for x dollars per item, B for y dollars per item. Express this total monthly revenue as a function of x and y.
Revenue = x * D1 + y * D2
Good Problem!

$$R(x, y) := x\,(50 - 10\, x + y) + y\,(20 + 3\, x - 5\, y)$$

$$\textit{Answer: R(x,y)} = 50\, x - 10\, x^2 + 4\, x\, y + 20\, y - 5\, y^2 \quad (49)$$

2008 47
Compute the indicated function value: if f(s,t) :=(t/s) + (s/t).
then f(6, –5) =
A) –2.03
B) 0.37
C) –1,830.00
D) –1.03

$$f(s,t) := \frac{t}{s} + \frac{s}{t}$$

$$f(4,-1) := -\frac{17}{4} \quad (50)$$

2008 48. SADDLE D < 0
48. Suppose z = f(x, y), f_x = f_y =0, f_xx = 5, f_yy = 1and f_xy=4 . Then (a, b) is
A) a relative minimum
B) a relative maximum
C) a saddle point
D) the test is inconclusive
Recall DISC:= f_xx * f_yy -f_xy * f_xy;
DISC = -11 SADDLE

$$f\_xx := 5$$
$$f\_yy := 1$$
$$f\_xy := 4$$
$$DISC := -11 \quad (51)$$

2008 49. Evaluate .

$$Integral := \int (5x^3 - 3x + 4)\, dx$$
$$Answer := \frac{5}{4}x^4 - \frac{3}{2}x^2 + 4x + C \quad (52)$$

2007 50
Find the function whose tangent line has the slope for each value of x and whose graph passes through (0, 2).

$$F(x) := 2 + \int_0^x e^s\, ds$$
$$f(x) := 1 + e^x$$
$$Answer\ y := 1 + e^x \quad (53)$$

2008 51.
NO CORRECT ANSWER CHECK ANSWER AInn object is moving so that its velocity after t minutes is
5 + 8*t + 9*t^2 meters per minute. How far does the object travel from the end of minute 4 to the end of minute 5?
72 meters
131 meters
–67 meters
–47 meters
Minute 4: t from 3 to 4; End of minute 4: t = 4

$$ANS := \int_4^5 (5 + 8t + 9t^2)\, dt$$
$$ans := 224 \quad (54)$$

2008 52. Just multiply through and use the power rule NOT a SUBSTiTUTION problem

$$f(x) := x^3 (2x + 5 - 3\sqrt{x})$$
$$ANS := \int x^3 (2x + 5 - 3\sqrt{x})\, dx$$
$$ans := -\frac{2}{3}x^{9/2} + \frac{2}{5}x^5 + \frac{5}{4}x^4 + C \quad (55)$$

2008 53

$$f(x) := \frac{1}{4x}$$
$$ANS := \int \frac{1}{4x}\, dx$$

$$ans := \frac{1}{4}\ln(x) + C \quad (56)$$

2007 54.

$$f(x) := \left(\frac{1}{3}x-9\right)^{2/3}$$

$$integral := \int \left(\frac{1}{3}x-9\right)^{2/3} dx$$

```
Creating problem #1

Applying substitution x = 27+3*u, u = 1/3*x-9 with dx = 3*du,
du = 1/3*dx
```

$$\int \left(\frac{1}{3}x-9\right)^{2/3} dx = \int 3u^{2/3}\, du$$

```
Reverting substitution using u = 1/3*x-9
```

$$\int \left(\frac{1}{3}x-9\right)^{2/3} dx = \frac{9}{5}\left(\frac{1}{3}x-9\right)^{5/3} \quad (57)$$

2008 55.

$$f(x) := 7e^{2x}$$

$$ANS := \int 7e^{2x}\, dx$$

$$ans := \frac{7}{2}e^{2x} + C \quad (58)$$

2008 56.

$$f(x) := (3x-5)^4$$

$$ANS := \int_{-1}^{3} (3x-5)^4\, dx$$

$$ans := \frac{11264}{5}$$

$$answer := 2252.8 \quad (59)$$

2008 57. About UNITS
Suppose the marginal cost is C'(x) = e^(-0.1x) , where x is measured in units of 300 items and the cost is measured in units of $5,000. Find the cost corresponding to the production interval [300, 1,800] .
A) $241
B) $72
C) $804
D) $158

$$MC(x) := e^{-0.1x}$$

$$5000 \int_{1}^{6} e^{-0.1\,x}\,dx$$

$$ans := 17801.28910 \tag{60}$$

2008 58.

$$Intf := -7$$

$$Intg := 3$$

$$ans := -10 \tag{61}$$

2008 59

$$f(x, k) := x^3 - 3\,x + k$$

$$AVG := \frac{1}{4} \int_{1}^{5} \left(x^3 - 3\,x + k\right)\,dx$$

$$avg := 30 + k$$

$$ans\ k := 20 \tag{62}$$

2008 60 NOT IN 2009 (CHECK JL) a good problem anyway (JL)
Find the consumers surplus for a commodity whose demand function is
D(q) = 30e-0.03q dollars per unit if the market price is 0 p = $21 per unit. (Hint:
Find the quantity q0 that corresponds to the given price p0 = D(q0).)
A) $49.53
B) $49.81
C) $50.33
D) $53.41

$$Dem := \mathbf{proc}(q)\ 30 * \exp(-0.03 * q)\ \mathbf{end\ proc}$$

$$Dem(q) := 30\,e^{-0.03\,q}$$

$$CS := q \rightarrow \int_{0}^{q} Dem(s)\,ds - q\,Dem(q)$$

$$q\_0 := 11.88916480$$

$$ans := 50. \tag{63}$$

2008 61 NO CORRECT ANSWER CHECK ANSWER

$$\int_{0}^{2} 1100\,e^{0.12 - 0.06\,t}\,dt$$

$$ans := 2337.44 \tag{64}$$

2008 62

$$\int_{0}^{14} 1100\,e^{0.56 - 0.04\,t}\,dt$$

$$ans := 20643.5 \tag{65}$$

2008 63.Good problem but NOT IN 2009
Demographic studies conducted in a certain city indicate that the fraction of the residents that will

remain in the city
for at least t years is
f(t) = exp(-.t/50). . The current population of the city is 500,000, and it is estimated that new residents will be a
rriving at the rate of 1,000 people per year. Assuming this estimate is correct, give an expression for the population as a function of t.

2008 Problem 63.. different units
Fix T. Of the original 500,000 there will remain 500000exp(-T/50).
Between t and t + dt, 1000 dt will arrive; of these, 1000 dt exp(-(T-t)/50 will remain.at time T
Let FP be the "future poulation" at time T from the "stream" arriving.

$$FP(T) := \int_0^T 1000\, e^{-\frac{1}{50}T + \frac{1}{50}t}\, dt$$

$$-50000\, e^{-\frac{1}{50}T} + 50000$$

$$Total\ Population := 450000\, e^{-\frac{1}{50}T} + 50000 \tag{66}$$

2008 Problem 64

$$f\_64(x,y) := 7\, x\, y^8$$

$$ans\_64 := 7\, y^8 \tag{67}$$

2008 Problem 65 - See hoffmannchap7 Problem 17

Problem 17 -- I won't use x since assigning \`x\` would require correction bfore proceeding to other problems.

$$Z := 9\, X - 8\, Y$$

$$X := t^3$$

$$Y := 11\, t \tag{68}$$

$$Answer\_17 := 27\, t^2 - 88 \tag{69}$$

$$Solution := \left(9 \Big|_{X = t^3,\ Y = 11\, t} \right) \left(\frac{d}{dt} \left(t^3\right) \right) + \left((-8) \Big|_{X = t^3,\ Y = 11\, t} \right) \left(\frac{d}{dt} (11\, t) \right)$$

$$Or,\ as\ we\ said := 27\, t^2 - 88 \tag{70}$$

Problem 66 is the same as hoffmann chapter 7 Problem 18

N.B. Numbers don't make much sense. Note that you start with negative profit!
18. A mall kiosk sells two different models of pagers, the Elite and the Diamond. Their monthly profit from pager sales is where x and y are the prices of the Elite and the Diamond respectively, in dollars. At the moment, the Elite sells for \$32 and the Diamond sells for \$40. Use calculus to estimate the change in monthly profit if the kiosk operator raises the price of the Elite to \$33 and lowers the price of the Diamond to \$38.
A) Profit will increase by about \$26.

B) Profit will decrease by about $310.
C) Profit will increase by about $194.
D) Profit will stay the same.
Construct the profit function P and us dP = P_x * dx + P_y * dy

$$P(x,y) := (x-40)\,(20-5\,x+6\,y) + (y-50)\,(30+3\,x-4\,y)$$

$$P(32,40) := -460$$

$$P\_x := 70-10\,x+9\,y$$

$$P\_y := 9\,x-10-8\,y$$

$$dP := (70-10\,x+9\,y)\Big|_{x=32,\,y=40} - 2\left((9\,x-10-8\,y)\Big|_{x=32,\,y=40}\right)$$

$$Answer\_18 := 194 \qquad (71)$$

2008 67. D>0 f_xx < 0 RELATIVE MAXIMUM

$$f\_xx := -5$$

$$f\_yy := -2$$

$$f\_xy := -3$$

$$\mathrm{D} := 1 \qquad (72)$$

2008 68. x^2 - y = -6 or y = x^2 + 6 is a parabola.

$$eqn := x^2 - y = -6$$

$$y := x^2 + 6$$

$$Ans := parabola \qquad (73)$$

2008 69.(Also in hoffmannchap7)
8. A soft drink can is a cylinder H cm tall with radius R cm. Its volume is given by the formula V = pi R^2 H.
A particular can is 8 cm tall with radius 1 cm. Use calculus to estimate the change in volume that results if the radius
is increased by 1 cm while the height remains at 8 cm.
A) The volume is increased by 32p cm .
B) The volume is increased by 16p cm .
C) The volume is increased by 1p cm .
D) The volume is increased by 8p cm
use dV = P_R * dR + P_H * dH; dH = 0. .

$$V(R,H) := \pi R^2 H$$

$$V\_R(R,H) := 2\,\pi\,R\,H$$

$$dV := (2\,\pi\,R\,H)\Big|_{R=1,\,H=8}$$

$$Answer\_8 := 16\,\pi \qquad (74)$$

2008 70 is in hoffmannchap7.mw
hoffmannchapter7.11. Daily output Q = 10 * K^(1/3) * L^(1/2) units.
Use marginal analysis to estimate the change in daily output as a result of changing L from 625 to 626 while K remains constant at 216.

$$Q(K,L) := 10\,K^{1/3}\sqrt{L}$$

$$dQ := diff(Q(K,L),K)*\,dK + diff(Q(K,L),L)*dL$$

$$dQ := \frac{10}{3}\,\frac{\sqrt{L}\,dK}{K^{2/3}} + \frac{5\,K^{1/3}\,dL}{\sqrt{L}}$$

$$\left(\frac{10}{3}\,\frac{\sqrt{L}\,dK}{K^{2/3}} + \frac{5\,K^{1/3}\,dL}{\sqrt{L}}\right)\Bigg|_{K=216,\,L=625,\,dL=1,\,dK=0}$$

$$Answer\_11 := \frac{1}{125}\,216^{1/3}\sqrt{625}$$

$$Answer := \frac{6}{5} \tag{75}$$

Answer Key

1.	A
2.	D
3.	A
4.	B
5.	A
6.	A
7.	D
8.	B
9.	A
10.	A
11.	A
12.	A
13.	C
14.	B
15.	A
16.	A
17.	A
18.	C
19.	A
20.	B
21.	B
22.	B
23.	A
24.	B
25.	B
26.	C
27.	B
28.	A
29.	B
30.	A
31.	D

32.	A
33.	A
34.	C
35.	B
36.	A
37.	A
38.	A
39.	C
40.	B
41.	D
42.	C
43.	B
44.	D
45.	
46.	
47.	A
48.	A
49.	A
50.	B
51.	A
52.	D
53.	A
54.	C
55.	B
56.	C
57.	C
58.	A
59.	D
60.	C
61.	C
62.	D
63.	D
64.	A
65.	B
66.	C
67.	C
68.	A
69.	B
70.	1.2