exam02sample.mw
Maple 10 Worksheet for Problems in Math 165 - Calculus for Business.
Answer Key for exam02sample
First load plots and student:

```
> restart: with( student):with (plots):with(plottools):
```

N.B. A Maple command such as eval(f(x),x=2) is the instruction
``Evaluate f(2)'' or
``evaluate the function f(x) at x = 2.''

Problem 1

```
> f_1:= proc(x);
  x^5 - x^4;
  end proc:`f_1(x) `:=f_1(x);
> ddx_f_1(x):=diff(f_1(x),x);
> f_1_prime:=factor(%);
  crit_num:=[solve(ddx_f_1(x) = 0,x)];
```

$$f\_1(x) := x^5 - x^4$$

$$ddx\_f\_1(x) := 5\,x^4 - 4\,x^3$$

$$f\_1\_prime := x^3\,(5\,x - 4)$$

$$crit\_num := \left[\frac{4}{5}, 0, 0, 0\right] \tag{1}$$

Check the endpoints!

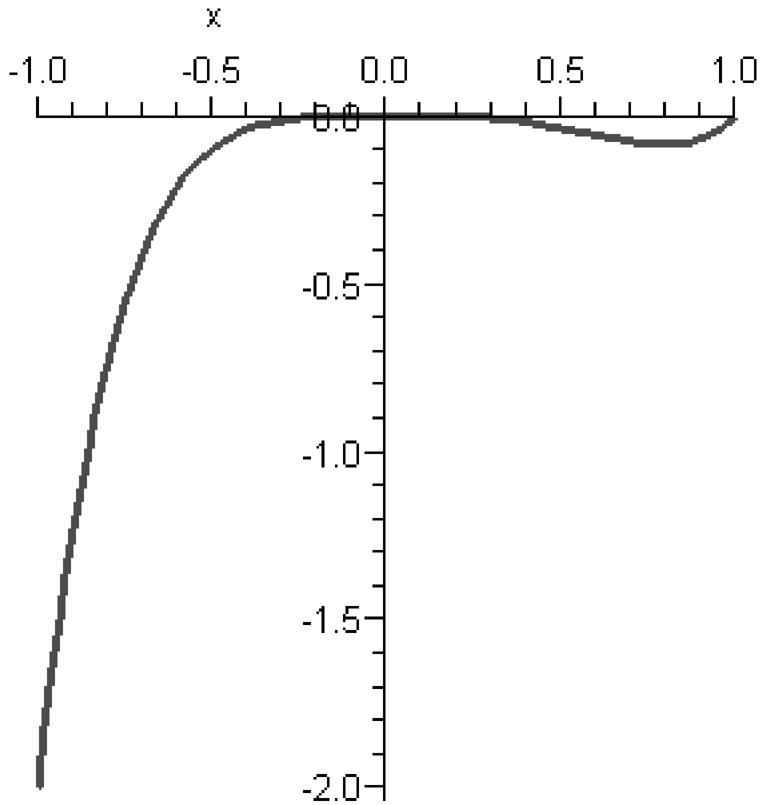
```
> f_crit_num:=[f_1(-1),f_1(0),f_1(4/5),f_1(1)];
```

$$f\_crit\_num := \left[-2, 0, -\frac{256}{3125}, 0\right] \tag{2}$$

The maximum value of f on the interval is 0, which occurs at x= 0 and at x = 1.

A graph for problem 1

```
> plot_1:=plot(f_1(x), x = -1 ..1, thickness = 2):display(plot_1);
```



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Problem 2

Solve f' = 0 to find all critical numbers

```
> f_2:= proc(x);
    2*x^2 - 8*x +7;
  end proc:`f_2(x) `:=f_2(x);
> d_dx_f_2(x):= diff(f_2(x),x);
> crit_num:=solve(% = 0,x);
```

$$f\_2(x) := 2x^2 - 8x + 7$$

$$d\_dx\_f\_2(x) := 4x - 8$$

$$crit\_num := 2 \tag{3}$$

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Problem 3

Solve f' = 0 to find critical numbers, check the sign of f' in between.

```
> f_3:=proc(x);
     4*x^3 + 18*x^2 - 120*x -4;
  end proc:`f_3(x) `:=f_3(x);
```

```
> d_dx_f_3(x):=diff(f_3(x),x);
> factored:=factor(%);
> int_increase:=solve(factored > 0,x);
> int_decrease:=solve(factored < 0,x);
  plot_3:=plot(f_3(x),x = -10 ..10, -1000 ..1000,thickness = 2,
  title = `y - range
  must be large`):display(plot_3);
>
```

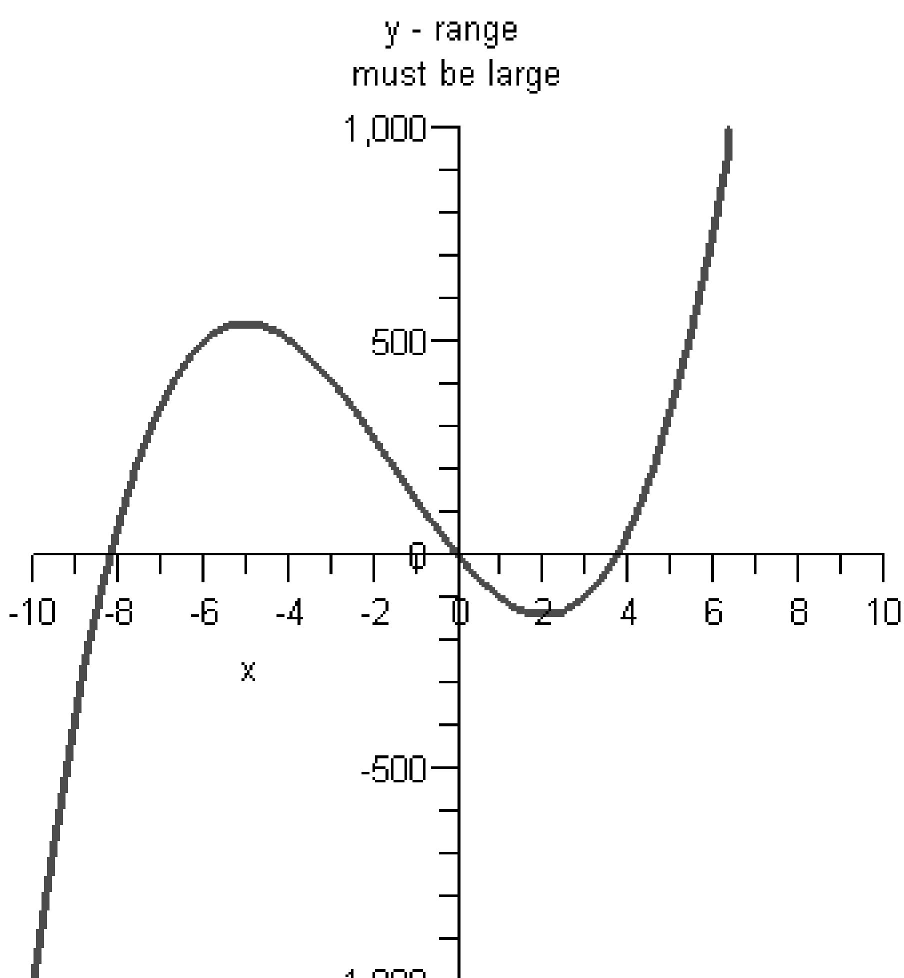
$$f\_3(x) := 4x^3 + 18x^2 - 120x - 4$$

$$d\_dx\_f\_3(x) := 12x^2 + 36x - 120$$

$$factored := 12\,(x+5)\,(x-2)$$

$$int\_increase := RealRange(-\infty, Open(-5)), RealRange(Open(2), \infty)$$

$$int\_decrease := RealRange(Open(-5), Open(2))$$



Problem 4

```
> f_4:=proc(x);
      4*x^2 - 6 * x + 1;
```

```
end proc: `f_4(x) `:=f_4(x);
f_4_prime:=proc(x);
   diff(f_4(x),x):
end proc; `f_4_prime(x) `:=f_4_prime(x);
crit_numbers:= solve(f_4_prime(x),x);
```

$$f\_4(x) := 4x^2 - 6x + 1$$

$$f\_4\_prime := \textbf{proc}(x)\ diff(f\_4(x), x)\ \textbf{end proc}$$

$$f\_4\_prime(x) := 8x - 6$$

$$crit\_numbers := \frac{3}{4} \tag{4}$$

****
Problem 5
Check for vertical asymptotes!

```
> f_5:=proc(x);
     5/(x^2 - 8*x +12);
  end proc:`f_5(x) `:=f_5(x);
  `vertical asymptotea at x `:=solve(x^2 - 8*x +12 = 0,x);
```

Vertical asymptotes at x= 2, 6.

$$f\_5(x) := \frac{5}{x^2 - 8x + 12}$$

$$vertical\ asymptotea\ at\ x := 6, 2 \tag{5}$$

Now solve f' = 0

```
> d_dx_f_5(x):= diff(f_5(x) ,x);
> crit_num:=solve(d_dx_f_4(x) =0,x);
```

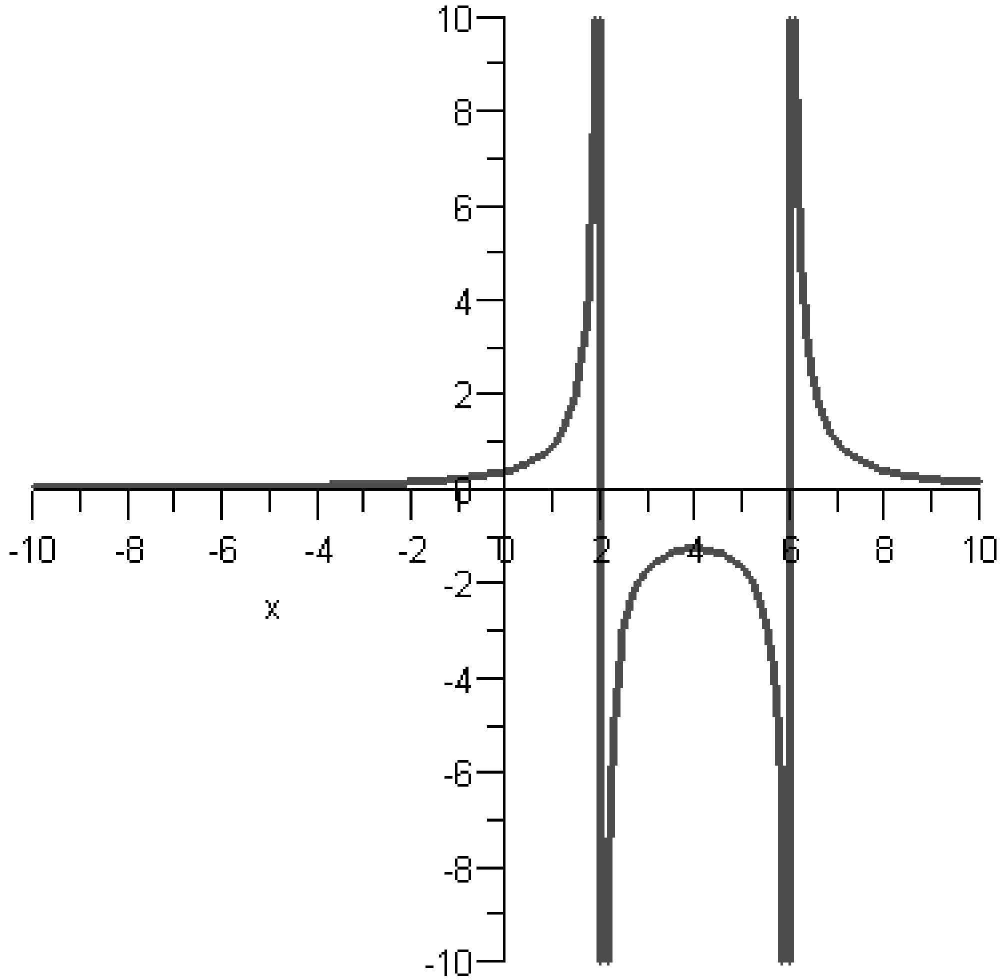
At x = 4, d_dx_f_4 changes from - to + so LOCAL MINIMUM

$$d\_dx\_f\_5(x) := -\frac{5\,(2x - 8)}{(x^2 - 8x + 12)^2}$$

$$crit\_num := \mathrm{RootOf}(d\_dx\_f\_4(\_Z)) \tag{6}$$

A graph for problem 5

```
> plot_5:=plot(f_5(x),x = - 10 .. 10,  -10 ..10, thickness = 2)
  :display(plot_5);
```



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Problem 6 ANSWER KEY MISTAKE
Profit = Revenue - Cost
Revenue(x) = x * (49 -x)

```
> R_6:= proc(x);
      x*(49 - x) - (x^2 + 4*x +7);
  end proc:`R_6(x) `:= R_6(x);
> MP_6(x):= diff(R_6(x) ,x);
> max_profit_x:=solve(MP_6(x)=0 , x);
```

$$R\_6(x) := x\,(49-x)-x^2-4\,x-7$$

$$MP\_6(x) := 45-4\,x$$

$$max\_profit\_x := \frac{45}{4} \tag{7}$$

****
Problem 7
Solve f' = 0. f' is a quadratic
Check sign of f''.
At a critical point, f'' > 0 says local maximum, f'' < 0 says local maximum.

```
> f_8(x):= 2*x^3 - 3 * x^2 - 12 *x +13;
  d_dx_f_8(x):= diff(f_8(x) ,x);
  crit_num:=solve(d_dx_f_8(x) = 0  , x);
  d2_dx2_f_8(x):= diff(f_8(x) ,x$2);
  f_prime_prime_crit_num:=
     [eval(d2_dx2_f_8(x),x= crit_num[1]),eval(d2_dx2_f_8(x),x=
  crit_num[2])];;
```

$$f\_8(x) := 2\,x^3 - 3\,x^2 - 12\,x + 13$$

$$d\_dx\_f\_8(x) := 6\,x^2 - 6\,x - 12$$

$$crit\_num := 2, -1$$

$$d2\_dx2\_f\_8(x) := 12\,x - 6$$

$$f\_prime\_prime\_crit\_num := [18, -18] \quad (8)$$

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Problem 8

Note: q = q(p) is a function of price.

N.B. Statement "14 more buzzers for every 12 cent decrease in price means
dq/dp = 14/(-0.12), and "price demand of elasticity" is
E(p) = (p/q)*(dq/dp)

```
Elasticity:= (1.40/60)*(14/(-0.12));
```

$$Elasticity := -2.722222223 \quad (9)$$

```
> q_8:=proc(p);   60 + (14/(-0.12))*(p - 1.40);end proc:
  `q_8(x) `:= q_8(x);

> P_8(p):= (p - .40)* q_8(p):`P_8(p) `:=P_8(p);
> d_dp_P_8(p):= diff(P_8(p) ,p);
> max_profit_price:=solve(d_dp_P_8(p)  , p);
```

$$q\_8(x) := 223.3333334 - 116.6666667\,x$$

$$P\_8(p) := (p - 0.40)\,(223.3333334 - 116.6666667\,p)$$

$$d\_dp\_P\_8(p) := 270.0000001 - 233.3333334\,p$$

$$max\_profit\_price := 1.157142857 \quad (10)$$

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Problem 9

Note Cost is a function of p

Profit(p) = p*(25 - p) - C(p)

```
> D_9(p):= 28 - 5 * p;
  P_9(p):=p*  D_9(p) - (p^2 + 4*p);
  d_dp_P_9(p):= diff( P_9(p),p);
  crit_num:=solve(d_dp_P_9(p)  ,p);
  max_profit_P:=eval(P_9(p),p=crit_num);
  evalb(max_profit_P = 24);
```

$$D\_9(p) := 28 - 5\,p$$

$$P\_9(p) := p\ (28 - 5\ p) - p^2 - 4\ p$$
$$d\_dp\_P\_9(p) := 24 - 12\ p$$
$$crit\_num := 2$$
$$max\_profit\_P := 24$$
$$true \qquad (11)$$

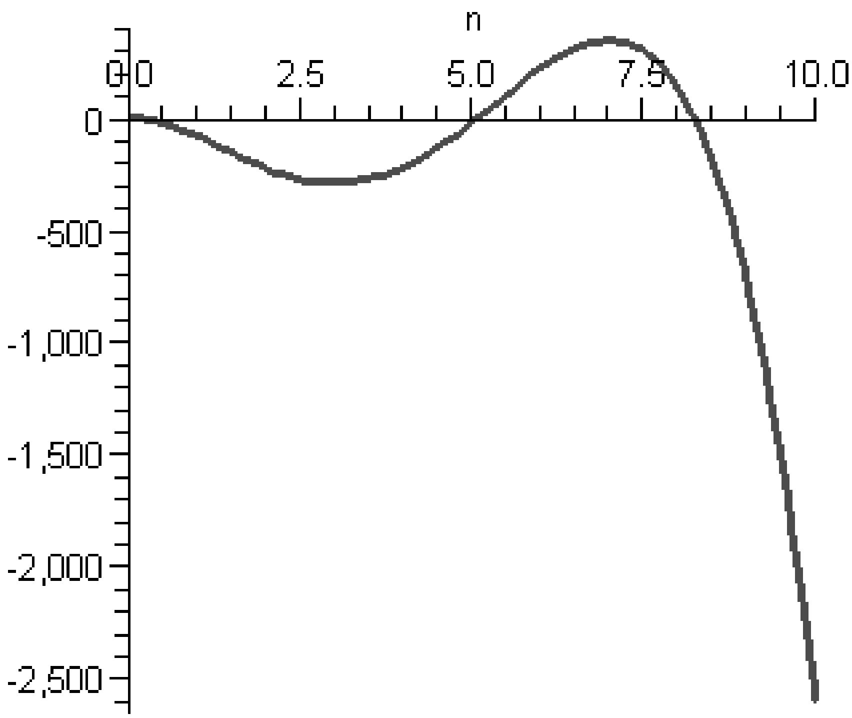
****
Problem (10): Note 0 <= n <= 10. Check the Endpoints!
Hiring the first 5 employees decreases net revenue!

```
> R_10:= proc(n):
    - 3*n^4 + 40*n^3 - 126*n^2 + 15;
  end proc:`R_10(n) `:=R_10(n);
  d_dn_R_10(n):= diff(R_10(n) ,n);
  crit_num:= solve(d_dn_R_10(n) = 0 , n);
  values_at_crit_end:=[eval(R_10(n),n = crit_num[1]),
  eval(R_10(n),n = crit_num[2]),
  eval(R_10(n),n = crit_num[3]),
  eval(R_10(n),n = 10)];
```

$$R\_10(n) := -3\ n^4 + 40\ n^3 - 126\ n^2 + 15$$
$$d\_dn\_R\_10(n) := -12\ n^3 + 120\ n^2 - 252\ n$$
$$crit\_num := 0, 7, 3$$
$$values\_at\_crit\_end := [15, 358, -282, -2585] \qquad (12)$$

Maximum 358 is at n = 7
A Gra.ph for Problem 10

```
> plot(R_10(n), n = 0  .. 10, thickness = 2);
```



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Problem 11

P_10:= 19000*exp(-.06*10)

```
> P_10:= 19000*exp(-.06*10);
```

$$P\_10 := 10427.42109 \quad (13)$$

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Problem 12 f(t):= f(0) * (f(1)/f(0))^t

```
> f_12(t):= 1 * (7/1)^t;
  `f(3) `:=eval(f_12(t),t=3);
```

$$f\_12(t) := 7^t$$

$$f(3) := 343 \quad (14)$$

****

Problem(13) P(0):= P(t) * exp(- r*t)

```
>  P_10:= 5000*exp(-.07*10);
  evalb(abs(P_10 - 2000) < 1);
```

$$P\_10 := 2482.926519$$

$$\mathit{false} \quad (15)$$