samples/hoffmannsamplechap4.mw
Maple 9.5 Worksheet for Problems in Math 165 - Calculus for Business.

First load plots and student:

```
> restart: with( student):with (plots):
```

N.B. A Maple command such as eval(f(x),x=2) is the instruction
``Evaluate f(2)" or
``evaluate the function f(x) at x = 2."

Problem 1 P(t) = P(0) e^(.10t)

```
> P1(t):= 3000*exp(.10*t);
  `ans_1 `:=eval(P1(t),t=9);
```

$$P1(t) := 3000\, e^{0.10\,t}$$

$$ans\_1 := 7378.809333 \tag{1}$$

Problem 2 B(0) = B(t) exp(-r*t)

```
> B1_0:= 14000 * exp(-0.05 * 40);
```

$$B1\_0 := 1894.693965 \tag{2}$$

Problem 3 P(0) = P(t) e^(-r*t). N.B. Answer in percent, NOT dollars

```
> eqn3:= 5000 = 2500 * exp(r*12);
  logeq3:=log(op(1,%))=log(op(2,%));
  simplify(%);
  solve (eqn3,r);
  `ans_3 r =`:= 100*evalf(%,3);
>
```

$$eqn3 := 5000 = 2500\, e^{12\,r}$$

$$logeq3 := \ln(5000) = \ln\left(2500\, e^{12\,r}\right)$$

$$3\ln(2) + 4\ln(5) = 2\ln(2) + 4\ln(5) + \ln\left(e^{12\,r}\right)$$

$$\frac{1}{12}\ln(2)$$

$$ans\_3\ r = := 5.7700 \tag{3}$$

Problem 4
Thinking about it - in (10 - 3) = 7 weeks, multipy by 1/2 - In 28 weeks we have the fraction (1/2)^4 of the bulbs left.

```
> ans_4:= (1/2)^(28/(10 - 3));
```

$$ans\_4 := \frac{1}{16} \tag{4}$$

Problem 5
The statement f(x) = e^{kx} says that f is an expontial growth function and f(0) = 1. Multiply by 10 every time x changes by 1.
f(x) = e^{kx}, f(1) = 10 = e^{k}. f(x) = 10^{x}.

```
> f_5(x):= 10^x;
  `f_5(3) `:=eval(f_5(x),x=3);
```

$$f\_5(x) := 10^{x}$$

$$f\_5(3) := 1000 \tag{5}$$

Problem 6.

f6(t) in billions. Multply by $\frac{5\ billion}{2\ billion}$ every 2 years. In 2010, t = 12 years from 1998;
Alternately, t = 12 years from 1998.

```
> f6(t):= (2)*(5/2)^(t/2);
  Ans_6_billions:=eval(f6(t),t=2010.-1998);
```

$$f6(t) := 2\left(\frac{5}{2}\right)^{\frac{1}{2}t}$$

$$Ans\_6\_billions := 488.2812500 \tag{6}$$

Problem 7

For true/false we use the Boolean evaluation of a statement.

```
> B(t):= A_0 * 2^(t/7);
  `After 14 years, B `:=eval(B(t),t=14);
  Ans_7:=evalb(%= 4*A_0);
```

$$B(t) := A\_0\, 2^{\frac{1}{7}t}$$

$$After\ 14\ years,\ B := 4\ A\_0$$

$$Ans\_7 := true \tag{7}$$

Problem 8

If g(x) = A e^(kx) = 20 -f(x), f(x) = 20 - g(x).
The given conditions are g(0) = 15 and g(3) = 17.
Thus g(x) = 15 (17/15)^(x/3), f8(x) = 20 - g(x).
If we use Boolean evaluaion, allow some error.

```
> g8(x):= 15*(17/15)^(x/3.);
  f8(x):= 20 - g8(x);
  f8(4):=eval(f8(x),x=4);
  evalb(abs(% - 0.7333) < .01);
```

$$g8(x) := 15\left(\frac{17}{15}\right)^{0.3333333333x}$$

$$f8(x) := 20 - 15\left(\frac{17}{15}\right)^{0.3333333333x}$$

$$f8(4) := 2.27573879$$

$$false \tag{8}$$

Problem 9

```
> B9(t):=A* (2)^(t/6);
  `B9(12) `:=eval(B9(t),t=12);
```

```
Ans_9:= evalb(% = 2*A);
```

$$B9(t) := A\,2^{\frac{1}{6}t}$$

$$B9(12) := 4\,A$$

$$Ans\_9 := false \tag{9}$$

Problem 10

```
> B10(t):= 800*(600/800)^(t/100);
  eval(B10(t),t=400.);
```

$$B10(t) := 800\left(\frac{3}{4}\right)^{\frac{1}{100}t}$$

$$253.1250000 \tag{10}$$

Problem 11

The form of the "machine answer" is thesame as $\frac{\left(1+\frac{\ln(b)}{\ln a}\right)}{2}$.

```
> eqn11:= a^(2*x - 1) = b;
  Ans_11:=solve(eqn11,x);
```

$$eqn11 := a^{2x-1} = b$$

$$Ans\_11 := \frac{1}{2}\ \frac{\ln(a)+\ln(b)}{\ln(a)}$$

Problem 12

```
> eqn12:= 2* ln(x) - (1/3)* ln(x^2) = 4;
  #LHS:=op(1,eqn12);RHS:=op(2,eqn12);
  #exp_eqn12:=e^(RHS)=e^ln(RHS);
  Ans_12:=solve(eqn12,x);
```

$$eqn12 := 2\ln(x) - \frac{1}{3}\ln\left(x^2\right) = 4$$

$$Ans\_12 := e^3$$

Problem 13 Fraction Working(t) = $e^{-0.25t}$.

Fraction Failing = 1 - Fraction Qorking

```
> W13(t):= exp(-0.25*t);
  Ans_13:= eval(1 - W13(t), t = 2);
```

$$W13(t) := e^{-0.25t}$$

$$Ans\_13 := 0.3934693403 \tag{11}$$

Problem 14

```
> f14(x):= x * exp(x);
  f14_prime(x):=diff(f14(x),x);
  y_0:=eval(f14(x),x = 1);
  slope:=eval(f14_prime(x),x=1);
```

```
Ans_14:= y = y_0 + slope * (x-1);simplify(%);
```

$$f14(x) := x\,e^{x}$$

$$f14\_prime(x) := e^{x} + x\,e^{x}$$

$$y\_0 := e$$

$$slope := 2\,e$$

$$Ans\_14 := y = e + 2\,e\,(x-1)$$

$$y = e\,(2x-1) \tag{12}$$

Problem 15

Differentiate the function

```
> diff(x^3 * exp(-3*x),x);
> Ans_15:=simplify(%);
```

$$3x^{2}e^{-3x} - 3x^{3}e^{-3x}$$

$$Ans\_15 := -3x^{2}e^{-3x}(x-1)$$

Problem 16

Differentiate the function; slope = ddx_f evaluated at x=2; point slope form

```
> f16(x):= exp(x^2);
  ddx_f16(x):=diff(f16(x),x);
  y_0:=eval(f16(x),x=2);
  slope_m:=eval(ddx_f16(x),x=2);
  Ans_16:= y = y_0 + slope_m * (x - 2);
  simplify(%);
```

$$f16(x) := e^{x^2}$$

$$ddx\_f16(x) := 2x\,e^{x^2}$$

$$y\_0 := e^{4}$$

$$slope\_m := 4\,e^{4}$$

$$Ans\_16 := y = e^{4} + 4\,e^{4}(x-2)$$

$$y = e^{4}(-7 + 4x)$$

Problem 17

```
> diff(ln(x^5),x);
```

$$\frac{5}{x}$$

Problem 18

Set up the Profit function, differentiate wrt x (price) and find the critical number

```
> Profit18(x):=
      (x - 10) * (200 * exp(-0.2 * x));
```

```
ddx_Profit18:=  proc(x)
 diff(Profit18(x),x);
end proc:
`Derivative of Profit `:=ddx_Profit18(x);

Ans_18:=solve(ddx_Profit18(x) = 0,x);
```

$$Profit18(x) := 200\ (x-10)\ e^{-0.2\,x}$$

$$Derivative\ of\ Profit := 200\ e^{-0.2\,x} - 40.0\ (x-10)\ e^{-0.2\,x}$$

$$Ans\_18 := 15.$$

Problem 19:
Marginal Cost of 14th === Derivative of Cost wrt level of production
-- **evaluated at 14 - 1**-- (CONVENTION)

```
> Cost19(x):=
     190 + 36*ln(6*x - 3);
  MC19(x):=
     diff(Cost19(x),x);
> Ans_19:=eval(MC19(x),x= 13.);
  Ans_19_14:=eval(MC19(x),x= 14.);
```

$$Cost19(x) := 190 + 36\ln(6\,x-3)$$

$$MC19(x) := \frac{216}{6\,x-3}$$

$$Ans\_19 := 2.879999999$$

$$Ans\_19\_14 := 2.666666666$$

Problem 20.

```
> Ans_20:=diff(exp(-6*x),x);
```

$$Ans\_20 := -6\ e^{-6\,x}$$

Problem 21

```
> f21(t):=A*exp(.07*t);
  ddt_f21(t):=diff(f21(t),t);
  percent21:=100*(ddt_f21(t)/f21(t));
```

$$f21(t) := A\ e^{0.07\,t}$$

$$ddt\_f21(t) := 0.07\ A\ e^{0.07\,t}$$

$$percent21 := 7.00$$

Problem 22

```
> f22(t):=A*exp(.12*t);ddt_f22(t):=diff(f22(t),t);
  rate22:=100*(ddt_f22(t)/f22(t));
```

$$f22(t) := A\ e^{0.12\,t}$$

$$ddt\_f22(t) := 0.12\ A\ e^{0.12\,t}$$

$$rate22 := 12.00$$

Problem 23

```
> f23(x):=exp(-3/(x+1));ddx_f23(x):=diff(f23(x),x);
> evalb(f23 = ddx_f23);
```

$$f23(x) := e^{-\frac{3}{x+1}}$$

$$ddx\_f23(x) := \frac{3\, e^{-\frac{3}{x+1}}}{(x+1)^2}$$

$$false$$

Problem 24

```
> f24(x):= x^x;
  f24_prime(x):= diff(x^x,x);
  Ans_24:=evalb(f24_prime(x) = x^x * (ln(x)));
> evalb(diff(x^x,x) = x^x * (ln(x)));
```

$$f24(x) := x^x$$

$$f24\_prime(x) := x^x\,(\ln(x) + 1)$$

$$Ans\_24 := false$$

$$false$$

Problem 25

```
> f25(x):= 4*x^4 - 80*ln(x);
> ddx_f25(x):= diff(f25(x),x);
> solve(ddx_f25(x)=0,x);
> crit25:=%[1];
> eval(f25(x),x=crit25 );
  minimize(f25(x),x = 0 .. infinity, location);
```

$$f25(x) := 4\,x^4 - 80\ln(x)$$

$$ddx\_f25(x) := 16\,x^3 - \frac{80}{x}$$

$$5^{1/4},\ I\,5^{1/4},\ -5^{1/4},\ -I\,5^{1/4}$$

$$crit25 := 5^{1/4}$$

$$20 - 20\ln(5)$$

$$20 - 20\ln(5),\ \{[\{x = 5^{1/4}\},\ 20 - 20\ln(5)]\}$$

Problem 26

```
> ddx_f26(x):=diff(exp(x),x);
```

$$ddx\_f26(x) := e^x$$

Problem 27

Be careful! - the function ln(2x) is defined only for x>0.

```
> d2dx2_f27(x):=diff(ln(2*x),x$2);solve({% < 0, x > 0},x);
```

Conclude that second derivative is NEGative and function is CONCAVE DOWN

$$d2dx2\_f27(x) := -\frac{1}{x^2}$$

$$\{0 < x\}$$

Problem 28

```
> ddx_f28(x):=diff(exp(2*x),x);
```

$$ddx\_f28(x) := 2\,e^{2x}$$