Solitary-wave Solutions of Systems of Nonlinear Wave Equations

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ABSTRACT

Discovery of solitary waves by John Scott-Russell in 1834 led to the Korteweg-de-Vries equation

$$u_t + u_x + uu_x + u_{xxx} = 0.$$

The numerical results in 1965 revealed the "soliton" or exact interaction phenomenon. These works indicated that any initial disturbance eventually resolves into a train of solitary waves. The inverse-scattering theory (1970s) confirmed the truth of the conjectures coming from numerical simulations and laboratory experiments. Solitary waves were thereby understood to play a fundamental particle role in the evolution of nonlinear, dispersive wave equations. This is why we are always interested to know whether a model describing wave propagation possesses solitary waves. Explicit solitary-wave solutions have been found for only a very limited number of equations. In most situations, we don't have such luck. So various theories have been developed to resolve this issue using nonlinear analysis. For example, in this talk, the topological degree theory and the concentration-compactness principle are applied to investigate existence of solitary-wave solutions of a general system of nonlinear, dispersive wave equations

$$\vec{w}_t + \vec{w}_x + \vec{f}(\vec{w})_x + \mathcal{L}\vec{w}_x - \mathcal{M}\vec{w}_t = 0,$$

where $\vec{w} = (u, v)$ is a vector of two component functions defined for $x \in R$ and $t \in [0, \infty)$, $\vec{f} = (f_1, f_2)$ consists of two nonlinear polynomials, the operators $\mathcal{L}$ and $\mathcal{M}$ are two matrices consisting of linear pseudo-differential operators as entries.