Katharine Ott: The mixed problem in Lipschitz domains.

Abstract. In this talk I will discuss the mixed problem, or Zaremba's problem, in a bounded Lipschitz domain. Consider $\Omega \subset \mathbb{R}^n$, $n \geq 2$, a bounded Lipschitz domain with boundary $\partial\Omega$ decomposed as $\partial\Omega = D \cup N$, with D and N disjoint and D an open subset of $\partial\Omega$. We specify Dirichlet boundary data on D and Neumann boundary data on the remainder of $\partial\Omega$. The problem reads

$$\mathcal{L}u = 0 \text{ in } \Omega, \quad u = f_D \text{ on } D, \quad \frac{\partial u}{\partial \nu} = f_N \text{ on } N.$$
 (0.1)

Above, \mathcal{L} is a second order elliptic elliptic operator with constant coefficients and ν is the outward unit normal vector defined a.e. on the boundary. We seek conditions on the domain, the boundary, and the data which guarantee that the gradient of the solution of (0.1) lies in $L^p(\partial\Omega)$ for $1 \leq p < \infty$. I will discuss recent results of this nature when the underlying operator \mathcal{L} is the Laplacian or the Lamé system of elastostatics. This is joint work with Russell Brown and Justin Taylor.