MthT 430 2006 Midterm Assessment

I. Definitions

1. (10 points) Define (ϵ–δ): $\lim_{x \to a} f(x) = L$.

2. (10 points) Is the following a correct definition of limit? Explain your answer.

Definition (Maybe).

$$\lim_{x \to a} f(x) = L$$

means: For every $\delta > 0$, there is an $\epsilon > 0$ such that, for all x, if $0 < |x - a| < \delta$. then $|f(x) - L| < \epsilon$.

II. Examples

3. (10 points) Give an example of two functions f and g such that $f \circ g$ and $g \circ f$ have **different nonempty domains**. Be sure to specify domain (f), domain (g), domain $(f \circ g)$, and domain $(g \circ f)$.

4. (10 points) Give an example of a function $f(x)$ defined for all real numbers such that $\lim_{x \to 0} f(x)$ does not exist.

5. (20 points) Let

$$F(x) = \sqrt{x^2 - 1},$$
$$G(x) = \frac{1}{x}.$$

Describe:

- domain (F) and domain (G).
- domain $(F + G)$
- domain $(G \circ F)$
- domain $(F \circ G)$
- domain $\left(\frac{F}{G}\right)$
- domain $\left(\frac{G}{F}\right)$

III. Proofs

6. (15 points) Prove: If g is continuous at a, $g(a) \neq 0$, then there is a $\delta > 0$ for which $(a - \delta, a + \delta)$ is contained in the domain of $\frac{1}{g}$.

7. (15 points) Show, using only P1 – P12: For all numbers a, b,

$$-(a \cdot b) = (-a) \cdot b.$$

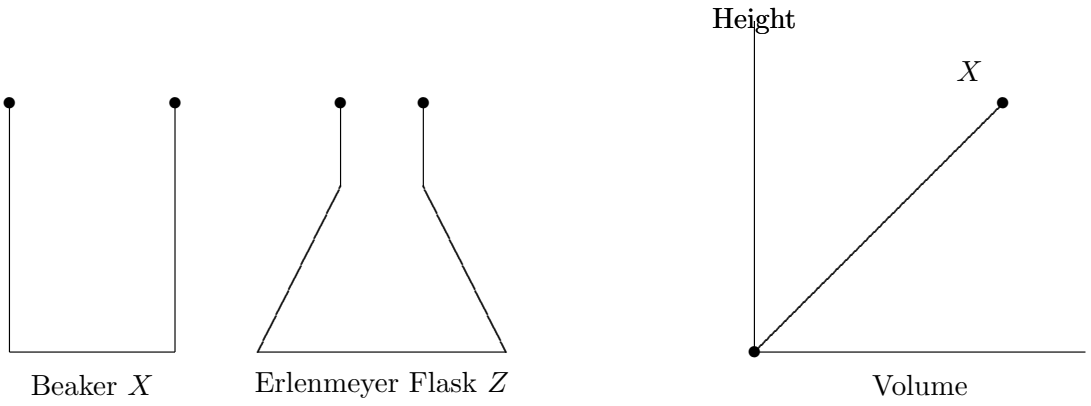
You may abbreviate (*distributive, trichotomy, ...*).

8. (15 points) Show by mathematical induction or otherwise: For all natural numbers $n = 1, 2, \ldots$,

$$1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

IV. Qualitative Properties of Functions

9. (25 points) The graph below shows how the height of a liquid in an Erlenmeyer Flask Z varies as water is steadily dripped into it. Copy the graph, and *on the same diagram* show the height–volume relationship for Flask Z.



Describe the features of the graph you have drawn. Your description should include

- The domain of the function
- The range of the function
- The intervals of monotonicity (Increasing, Decreasing)
- The intervals of constant concavity and/or linearity
- Other observations ...

A person reading your description of the graph should be able to reproduce the graph of the function (and if she's good, guess that it came from something shaped like an Erlenmeyer Flask).

V. Essay

10. (Letter Grade: A - E) In the exam booklet, write an essay on a topic of your choice that is very relevant to the material considered in the course. Your essay should include at least one substantial example and at least one substantial theorem and its proof.