MthT 430 Problem Set 11
In class November 14, 2007 – Turn in November 28, 2007

A typed paper is preferred, but a neat hand written paper is OK.

Group Work Rules:

- You are encouraged to work together!

- Away from the group, do your own neat write up of the problems.

- Acknowledge the group members and any other person/resource you use.

1. Let f be continuous on $[0, \infty)$ and differentiable on $(0, \infty)$; suppose that

$$\lim_{h \to 0+} \frac{f(0 + h) - f(0)}{h} \text{ exists.}$$

   Show that there is a function g, continuous and differentiable on $(-\infty, \infty)$, such that, for all $x \geq 0$, $g(x) = f(x)$.

2. Spivak Chapter 9, Problem 14.

   Let $f(x) = x^2$ if x is rational, and $f(x) = 0$ if x is irrational. Prove that f is differentiable at 0. (Don't be confused by this function. Just write out the definition of $f'(0)$.)

3. Spivak Chapter 9, Problem 22 (Part (b) modified).

(a) Suppose that f is differentiable at x. Prove that

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x - h)}{2h}$$

(B) Give an example of a function of a function f which is **not** differentiable at 0, but

$$\lim_{h \to 0} \frac{f(0 + h) - f(0 - h)}{2h} \text{ exists.}$$

4. Spivak Chapter 9, Problem 28.

(a) Find $f'(x)$ if $f(x) = |x|^3$. Find $f''(x)$. Does $f'''(x)$ exist for all x?

(b) (Statement slightly modified) Let $f(x) = x^4$ for $x \geq 0$ and $f(x) = -x^4$ for $x \leq 0$. Find $f'(x)$, $f''(x)$, and $f^{(3)}(x)$. Does $f^{(4)}(x)$ exist for all x?