MthT 430 Notes Chapter 5d Sequences and Limits

Sequences Cf. Spivak Chapter 22.

Definition. *An* **infinite sequence** *is a function whose domain is* **N**.

As a convention, we also allow the domain of a sequence to be a subset of **N** which includes all natural numbers *sufficiently large*.

Notation

If a is the name of the sequence, instead of listing the particular values by

$$a(1), a(2), \ldots,$$

we almost always use the subscript notation

$$a_1, a_2, \ldots.$$

We denote the sequence by

$$\{a_n\}$$

Limits of sequences

Definition. *A sequence $\{a_n\}$ converges to L (in symbols $\lim_{n\to\infty} a_n = L$) iff for every $\epsilon > 0$, there is a natural number N such that, for all natural numbers n,*

$$\text{if } n > N, \text{then } |a_n - L| < \epsilon.$$

A sequence $\{a_n\}$ is said to **converge** if it converges to L for some [finite!] number L, and to **diverge** if it does not converge.

Compare

- For a function f whose domain includes *all x sufficiently large and positive*,

$$\lim_{x\to\infty} f(x) = L.$$

- For a sequence $\{a_n\}$, whose domain includes *all n sufficiently large and positive*,

$$\lim_{n\to\infty} a_n = L.$$