MthT 430 Projects Chapter 7 Three Hard Theorems Turn In November 8, 2006

A typed paper is preferred, but a neat hand written paper is OK.

The point of the assignment is to show that all the Hypotheses in the Three Hard Theorems are necessary to assure the Conclusion. Construct (by pictures or formulas or a *very good* description) functions which violate one of the hypotheses and do not satisfy the conclusion.

1. Let $f(x) = |(1-x)\sin(1/x)|$, $0 < x \leq 1$, $f(0) = 0$. Sketch and describe the graph of f on $[0,1]$.

CFIVP: Continuous Functions on Intervals Have the Intermediate Value Property

2. Construct a function f on $[0,1]$ such that

- f is continuous on $[0,1]$ except at $x = \frac{1}{2}$,
- $f(0) > 0 > f(1)$,
- There is no $x \in [0,1]$ such that $f(x) = 0$.

CFCIB: Continuous Functions on Closed Intervals are Bounded

3. Construct a function f on $[0,1]$ such that

- f is continuous on $[0,1]$ except at $x = 1$,
- $f(0) < 0 < f(1)$,
- f is not bounded on $[0,1]$.

CFCIMAX: Continuous Functions on Closed Intervals assume a Maximum Value for the Interval

4. Construct a function f on $[0,1]$ such that

- f is continuous on $[0,1]$ except at $x = \frac{1}{2}$,
- f is bounded on $[0,1]$,
- $f(0) = 0 = f(1)$,
- There is no number y in $[0,1]$ such that $f(y) \geq f(x)$ for all x in $[0,1]$,
- There is no number y in $[0,1]$ such that $f(y) \leq f(x)$ for all x in $[0,1]$.