

Blocks R Us

Rough Draft due Feb 6

Math 300, Spring 2019, Schneider

This is a 4 to 6 page essay assignment. The first draft is due on **Oct 3**. You must meet with me in my office for an hour, twice, to work on your draft.

In this essay you will describe some counter-intuitive properties of infinite sets for a lay audience, and give an informal introduction to convergent and divergent series.¹ Think of this essay as an article that might appear in popular magazine devoted to science or as an article in a Sunday paper. You may assume your audience has a background in precalculus mathematics but no calculus. Emphasize pictorial arguments or “picture proofs”.

The Blocks Unlimited store sells various sets of toy blocks. One set, called the Deluxe Set, consists of infinitely many cubes, the first of which is 1 ft. by 1ft. by 1ft., the second cube has edge length $1/2$ ft, the third has edge length $1/3$ ft, and the n th cube has edge length $1/n$ ft. A second set of blocks, called the Starter Set, is a subset of the Deluxe Set. It consists of infinitely many cubes, the first of which has edge length 1 ft, the second has edge length $1/2$ ft, the third has edge length $1/4$ ft, and the n th cube in the Starter Set has edge length $1/2^{n-1}$ ft.

Give a convincing argument that if all the blocks in the Deluxe Set were stacked one on top of the other, then the stack would extend beyond the orbit of the moon but that it is possible to pack the Deluxe Set into a box that would be small enough to easily fit inside the trunk of a sports car. Since the Starter Set is a subset of the Deluxe Set, it could be packed in the same box used for the Deluxe Set. Argue that if the cubes in the Starter Set were stacked one on top of another, then the stack would be not very high at all.

The cubes in the Deluxe Set are sold unpainted. The Blocks Unlimited store also sells a special paint that can be used to paint these cubes. The paint is special because it has zero thickness. This paint is sold by the square foot. Estimate how many square feet of paint one would need to buy in order to paint all the faces of all the cubes in the Starter Set. How about the Deluxe Set?

¹This prompt is borrowed, with permission, from professors Baldwin and Thulin, who based it on assignments of Berman and Radford, who were in turn inspired by an exercise in *Writing in the teaching and learning of mathematics*, J. Meier and T. Rishel, MAA 1998.

(You may reference, but do not deeply explore, the the Basel Problem, Apery's constant, or other topics that are relevant but whose details exceed our scope.)

Each of the aforementioned quantities— the total height, surface area, and volume of each block-set— can be expressed as a series summation. Explore the idea of convergence and divergence of each series. (Many of these series can be shown to converge using the same picture!)

The primary purpose of this essay is to explain mathematics, and this should be made clear in your opening paragraph. However, please get as creative as you like with the narrative framing device (the “toy store” scenario). If you choose to weave your mathematical exposition into a story, keep the mathematics in the foreground throughout. The story exists only to motivate the math.