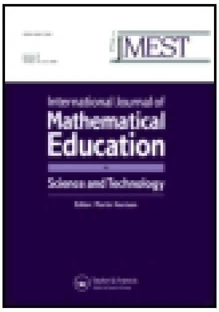

International Journal of Mathematical Education in Science and Technology
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tmes20

The card game 24 and its application to math education

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Published online: 20 Jan 2014.

To cite this article: Liping Tong, Jie Yang, Xue Han & Loren Velasquez , International Journal of Mathematical Education in Science and Technology (2014): The card game 24 and its application to math education, International Journal of Mathematical Education in Science and Technology, DOI: 10.1080/0020739X.2013.868544

To link to this article: http://dx.doi.org/10.1080/0020739X.2013.868544

International Journal of Mathematical Education in Science and Technology, 2014
http://dx.doi.org/10.1080/0020739X.2013.868544

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CLASSROOM NOTE

The card game 24 and its application to math education

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(*Received 29 May 2013*)

The card game 24 is a mathematical game that traditionally engages elementary students to practice their mental computational skills. In this paper, we use probability to formulate and explore the game. We create score of difficulty level for solvable card combination set under various setup of game rules. Based on our findings, we create new playing rules and provide suggestions for different levels of players. Our results may serve as guidelines on introducing the game 24 into the practice of math education.

Keywords: card game; solvable card combination set; probability of success; difficulty score; math education

1. Introduction

The history of the card game 24 can be traced back to Shanghai, China, in 1960s. By the end of the twentieth century it became known to North America. A standard deck of 54 poker cards may be used for the game 24. Typically, all the picture cards (King, Queen, Jack and Joker) are removed leaving only the 40 cards with values ranging from Ace (considered as 1) to 10. In each round of the game, four cards are randomly chosen. The goal is to develop a mathematical expression equal to 24 using all the four cards and four arithmetic operations (addition, subtraction, multiplication, and division). Each card has to be taken into account once and only once. Each arithmetic operation could be used more than once. Whoever finds an answer first wins this round of the game.

Depending on the four cards chosen a mathematical expression resulting in 24 may not exist. For example, there is no way to get 24 out of the four cards 1, 2, 9, and 10. When a solution does exist, it may or may not be unique. For example, for the cards 3, 5, 7, and 10, there are multiple answers leading to 24, including $(5 - 3) \times 7 + 10 = 24$, $(10 - 7 + 5) \times 3 = 24$ and $(3 + 5) \times (10 - 7) = 24$. If the four cards are 1, 5, 5, and 5, the only distinct answer is $(5 - 1 \div 5) \times 5 = 24$. Here, we regard $5 \times (5 - 1 \div 5) = 24$ as the same.

The traditional card game 24 has been used to engage young kids in practicing computation skills with whole numbers and mental calculation. The examples aforementioned ask students to draw on different math knowledge to come up with the result of 24. To

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successfully find the solutions from any chosen four cards, students need to be proficient with basic computation facts, the order of operations, and the basic number properties such as associative property, commutative property, distributive property, and identity property (multiplicative property). In addition, the example used above, $(5 - 1 \div 5) \times 5 = 24$, creates a fraction to find the answer 24, which expands and enhances student number sense development. For solving this problem, students may convert the fraction $\frac{1}{5}$ into decimal 0.2, calculate the subtraction result in mixed number, or apply distributive property to get $5 \times 5 - \frac{1}{5} \times 5$. The multiple ways to solve those similar problems can contribute to developing student number sense of whole numbers, fractions and decimals. A good number sense is the foundation for K-8 mathematics.[1] This is also an example that shows the connections among mathematics knowledge, which is one of the learning process standards that math educators need to foster in and demonstrate for students.[2] Mathematics teachers can also use the game to develop in their students two of the mathematics practices proposed by the Common Core State Standards for Mathematics [3] – Attending to Precision and Look for and Make Use of Structure. In this paper, we attempt to vary the rules of card game 24 that will take on more educational purposes of the traditional game. Teachers may choose appropriate difficulty levels of the game for different students by numbers of cards and arithmetic operations and use the game to explore the topics of probability.

It is natural to ask why the number 24 was of interest? Instead, can we require the target number to be some other number, such as 20? Do we have to use four cards? Can we use three or five cards? Can we restrict the arithmetic operations to addition and subtraction only to fit for younger kids? Can we use face cards (for example, Jack for 11, Queen for 12, and King for 13) as well for elder kids? The answer is Yes. We can make new rules and create new types of card games. To make the new game feasible and interesting, we need to make sure that our new rules can fit different levels of players and yield a high enough probability that has an answer for each round. In this paper, we aim to answer the above questions and give suggestions on choices of cards and arithmetic operations to different levels of players.

Assume that cards are well shuffled. Four cards are randomly selected, which might be (3,5,7,10), or (1,5,5,5), or (1,2,9,10), or some other card combinations. A solution leading to the target number, such as 24, may or may not exist. A card combination is called solvable if a solution exists. Otherwise, it is called impossible. We regard the card combination (7,3,5,10) the same as (3,5,7,10) since they do not make difference in answering our questions. In the section of METHODS, we show different ways to explore the set of distinct card combinations. We classify card combinations into two sets: solvable or impossible. We call it a success if a randomly selected card combination is solvable. In addition, some card combinations, such as (3,5,7,10), appear more often than others, such as (1,5,5,5). We use the frequency of each card combination as the weight in calculating the probability of success.

2. Methods

There are three steps in our calculation. First, we collect all distinct card combinations and let n denote the total number of them. In the second step, we check each distinct card combination whether it is solvable or impossible. Define $s_i = 1$ if the ith distinct card combination is solvable and $s_i = 0$ otherwise, where i ranges from 1 to n. In addition, for each distinct card combination, we calculate the frequency f_i and the difficulty score d_i that will be defined later, where $i = 1, \ldots, n$. Finally in the third step, the probability of success for a randomly chosen card combination is calculated by

$$P\,(\text{success}) = \frac{\sum_{i=1}^{n} s_i f_i}{\sum_{i=1}^{n} f_i} \tag{1}$$

and for the solvable card set the average difficulty score is

$$E\,(\text{difficulty}|\text{solvable}) = \frac{\sum_{i=1}^{n} s_i f_i d_i}{\sum_{i=1}^{n} s_i f_i}. \tag{2}$$

Based on these values, we can then make suggestions on rules of play and levels of players.

2.1. *Total number and list of distinct card combinations*

For illustration purpose, we use the combination of four cards ranged from A to 10 and four arithmetic operations (+, −, ×, ÷) as an example to show the procedure of our method. To find out the number of distinct card combinations, there are a couple of different ways. In [4] and [5], this problem is considered as being equivalent to choosing 4 x's from 13 positions (9 separators from 10 numbers plus 4 x's), though [5] gave solution to a more generalized problem. For example, the following diagram corresponds to the card combination (2,3,3,5).

1	2	3		4	5	6	7	8	9	10
	x	x	x		x					

Therefore, the total number of distinct card combinations is $n = \binom{13}{4} = 715$.

We may also calculate it as following. There are 5 different types of card combinations: (1) 4 distinct numbers, (2) 1 pair plus 2 distinct numbers, (3) 2 distinct pairs, (4) 1 triplet plus 1 distinct number, and (5) 1 quartet. The counts for these situations are $\binom{10}{4} = 210$, $\binom{10}{1}\binom{9}{2} = 360$, $\binom{10}{2} = 45$, $\binom{10}{1}\binom{9}{1} = 90$, and $\binom{10}{1} = 10$, respectively. Therefore, the total number of distinct card combinations is $210 + 360 + 45 + 90 + 10 = 715$. The way of classifying the distinct card combinations into five categories is useful when we calculate the frequencies of card combinations. Both this way and Triplett's approach (2011) find the correct answer. However, neither of them is ideal for programming and quick reference purposes.

In favour of convenient programming and quick reference, we come up with the third way to handle this problem. Start from the card combination (1,1,1,1) and increase by one at each time while keeping each number in the combination no less than any number to its left. That is, we construct a sequence of card combinations as follows: (1,1,1,1), (1,1,1,2), ..., (1,1,1,10), (1,1,2,2), (1,1,2,3), ..., (1,1,2,10), (1,1,3,3), (1,1,3,4), ..., (1,1,10,10), (1,2,2,2), (1,2,2,3), ..., (1,10,10,10), (2,2,2,2), (2,2,2,3), ..., (10,10,10,10). Let i_1, i_2, i_3, and i_4 represent the first, second, third and fourth number, respectively. We define a loop letting i_1 vary from 1 to 10, i_2 vary from i_1 to 10, i_3 vary from i_2 to 10 and i_4 vary from i_3 to 10. This loop ends up with 715 distinct card combinations with indices from 1 to 715. For example, the index is 1 for (1,1,1,1), 2 for (1,1,1,2), and 715 for (10,10,10,10). In addition, since these combinations are arranged alphabetically, given an arbitrary card combination, we can easily find its unique index. By this way, we construct a reference table with all distinct card combinations listed and ordered. For each card combination in the reference table, we only need to find out its solvability (s_i) and difficulty score (d_i) once. An extended reference table could also be constructed for A to King with a given set of arithmetic operations. The reference table will be especially useful for math educators or players seeking for help.

2.2. *Frequency of card combination*

A card combination, such as (3,5,7,10), may come from one or more suits. For example, it may contain a heart 3, a diamond 5, a club 7 and a spade 10, or hearts of all four numbers, or something else. For the ith card combination, we use frequency f_i to denote the total number of cases leading to the same card combination. For the 5 different situations: 4 distinct numbers, 1 pair plus 2 distinct numbers, 2 distinct pairs, 1 triplet plus 1 distinct number, and 1 quartet, the frequencies (f_i) are $4 \times 4 \times 4 \times 4 = 256$, $\binom{4}{2} \times 4 \times 4 = 96$, $\binom{4}{2} \times \binom{4}{2} = 36$, $\binom{4}{3} \times 4 = 16$ and 1, respectively. These frequencies are used in calculating the probability of success and the average difficulty score defined in formulas (1) and (2).

2.3. *Difficulty score*

We assign a score of 1 to the arithmetic operation addition (+), 2 to subtraction (−), 3 to multiplication (×), and 4 to division (÷). For a mathematical expression, we define the difficulty score to be the sum of scores for all arithmetic operations. For example, the score for $(3 + 5) \times 2 + 8$ is $1 + 3 + 1 = 5$; for $10 \times 10 \div 4 - 4$ is $3 + 4 + 2 = 9$. If a card combination is solvable with multiple solutions, we define its difficulty score to be the lowest score among all solutions. For example, the combination (4,5,6,7) has solutions $4 \times (5 + 7 - 6) = 24$, $4 \times (5 - (6 - 7)) = 24$, or $(6 - 4) \times (5 + 7) = 24$. The difficulty scores are 6, 7 and 6, respectively. Note that the first and second solutions are algebraically equivalent though the difficulty scores are different. There are many other algebraically equivalent solutions which we didn't list here since none of them has a score lower than 6. Therefore, the difficulty score for (4,5,6,7) is 6.

Note that the definition of difficulty score can be refined under more sophisticated considerations on the mathematical expressions. For example, since 1×8 is obviously easier than 9×8, different weights can be assigned according to different values of the two numbers involved in. In addition, it is easier to get an answer for a card combination with multiple solutions such as (3,5,6,8) than for a card combination with a unique solution such as (4,4,10,10). Nevertheless, the difficulty score is not our major concern in this paper. We prefer a simple definition for easy understanding.

3. Results

In this paper, we consider only the situations of three-card or four-card combination with two (+,−), three (+,−,×) or four (+,−,×,÷) arithmetic operations. The situation of five or more cards is much more complicated and not so useful in practice. As for target number, we consider not just 24 but all integers from 0 to 50 for comparison purpose.

3.1. *Three cards*

In this subsection, we explore the probability of success and score of difficulty when three cards are randomly selected from 40 (A to 10) or 52 (A to K) cards. For each

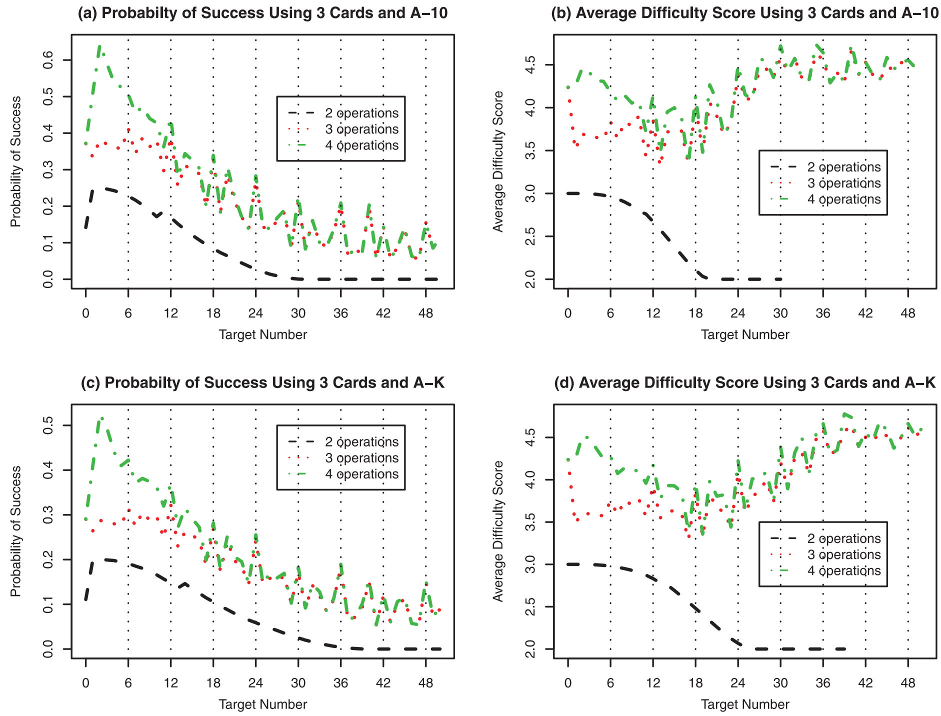

Figure 1. Probability of success and average difficulty score using three cards randomly selected from A to 10 or from A to K.

card combination, we allow two, three or four arithmetic operations. The results across different target numbers ranging from 0 to 50 are displayed in Figure 1. The patterns in Figure 1(a) and 1(c), or in Figure 1(b) and 1(d) are similar. From these figures, we see that the more operations allowed, the higher probabilities and more difficult on average to find a solution (black dashed curve at the bottom and green dash-dotted line on the top), which is reasonable. The two probability curves, red dotted and green dash-dotted in Figure 1(a) and 1(c), are almost the same for target numbers higher than 18. This is because when only three cards get involved in the calculation, the operation division ($\div$) is barely used to result in a target number of 18 or more. This is also verified by the two overlapped curves (red dotted and green dash-dotted) in Figure 1(b) and 1(d). When only two operations ($+,-$) are allowed, the probability of success and average difficulty score both decrease as the target number increases. However, when multiplication ($\times$) and division ($\div$) are involved, along with the increase of the target number, the probability of success decreases while the average difficulty score increases. It is interesting to see that for the red dotted and blue dash-dotted curves in Figure 1(a) and 1(c), the multiples of 6, that is, 6, 12, 18, etc., usually have the local maximum probabilities of success. The number 6 is the product of the two smallest prime numbers 2 and 3. Therefore, when multiplication ($\times$) is used, multiples of 6 are more likely to be solved.

When only two operations ($+,-$) and cards A to 10 are considered, for any single target number between 0 to 50 the probability of success is very low (less than 0.26, see the black dashed curve in Figure 1(a)). If we allow multiple target numbers and define the target number set as $\{1, 2, 3\}$, the total probability of success is 0.65 and the average difficulty score is 3. If we further extend the target number set to $\{1, 2, 3, 4, 5\}$, the total probability of success becomes 0.87 and the average difficulty score is still 3. When two operations $(+,-)$ and cards A to K are considered, the situation is similar: the probability of success is very low for any single target number (less than 0.21, see the black dashed curve in Figure 1(c)). Again define the target number set as $\{1, 2, 3\}$, then the total probability of success is 0.54 and average difficulty is 3. Extend the set of target numbers to $\{1, 2, 3, 4, 5\}$, then the total probability of success becomes 0.76 and the average difficulty is still 3. Apparently adding J, Q, and K into the game does not increase the probability of success or make it more challenging. Therefore, for the two-operation $(+,-)$ and three-card setup we suggest to use cards from A to 10 and allow multiple target numbers such as $\{1, 2, 3, 4, 5\}$. These results are also summarized in Table 2 in Section 'Discussion'.

Consider three operations $(+,-,\times)$. For cards from A to 10, the best target number is 6 since it has the highest probability of success (0.41) and relatively low average difficulty score (3.84). For cards from A to K the best choice is also 6, which has the highest probability of success (0.31) and low average difficulty score (3.75). Again, using cards A to K does not contribute to the probability of success or increase difficulty score. Our suggestion is to use cards from A to 10 and set the target number to 6 (Table 2).

When four operations $(+,-,\times,\div)$ are considered, if using cards from A to 10, the target number 2 has the highest probability of success (0.64) and relatively high average difficulty score (4.46); if using cards from A to K the number 2 also has the highest probability of success (0.53) and relatively high average difficulty score (4.52). Our suggestion is to use cards from A to 10 and set the target number to 2 (Table 2).

3.2. *Four cards*

When four cards are randomly selected from 40 (A to 10) or 52 (A to K) cards, we also analyze the results with two $(+,-)$, three $(+,-,\times)$, or four $(+,-,\times,\div)$ arithmetic operations. In addition, since decimals/fractions may appear in the four-operation situation, we separate the results into two categories: allowing and not-allowing decimals/fractions. Note that even in the previous three-card four-operation situation, decimals/fractions may also appear in some solutions. Actually, for any two integers a and b, a fraction can only occur as the result of division $a \div b$ or $b \div a$. Without loss of generality, assume it comes from $a \div b$. When there are only three cards, once a fraction $a \div b$ is generated the next operation must be multiplication with the third number c to get a final integer target number. Since $a \div b \times c = a \times c \div b$, a fraction/decimal can always be avoided by taking the solution $a \times c \div b$ instead. However, in some four-card situation, for example, $6 \div (5 \div 4 - 1) = 24$, a fraction cannot be avoided. Therefore, it is necessary to calculate the results separately for allowing and not-allowing decimals/fractions, which are referred as results in real number world and integer world, respectively.

For the case of four-card and two-operation situation, the results are similar to the three-card two-operation situation: there is no a single target number that has a reasonably high probability of success. The best choice would be allowing the target number to be any of 1, 2, 3, 4 and 5. In this situation, if using cards from A to 10 the probability of success is 0.97 and the average difficulty score is 4; if using cards from A to K, these two numbers are 0.91 and 4. By this way, both could be a fun game with high probability of success.

For the case of four-card and three-operation situation, if using cards from A to 10 the number 12 has the highest probability of success 0.8341 and moderate average difficulty score 5.43, the number 6 can also be a good choice with the second highest probability of success 0.8325 and slightly lower average difficulty score 5.33. Using cards from A to

Table 1. List of card combinations solvable only in real number world for four-card four-operation and target number 24.

No.	Card combination	Frequency	Solution
1	(1,3,4,6)	256	$6 \div (1 - 3 \div 4) = 24$
2	(1,4,5,6)	256	$6 \div (5 \div 4 - 1) = 24$
3	(1,5,5,5)	16	$(5 - 1 \div 5) \times 5 = 24$
4	(1,6,6,8)	96	$6 \div (1 - 6 \div 8) = 24$
5	(2,4,10,10)	96	$(4 \div 10 + 2) \times 10 = 24$
6	(2,5,5,10)	96	$(5 - 2 \div 10) \times 5 = 24$
7	(2,7,7,10)	96	$(10 \div 7 + 2) \times 7 = 24$
8	(3,3,7,7)	36	$(3 \div 7 + 3) \times 7 = 24$
9	(4,4,7,7)	36	$(4 - 4 \div 7) \times 7 = 24$
10	(1,8,Q,Q)	96	$12 \div (12 \div 8 - 1) = 24$
11	(2,2,J,J)	36	$(2 \div 11 + 2) \times 11 = 24$
12	(2,2,K,K)	36	$(2 - 2 \div 13) \times 13 = 24$
13	(2,3,5,Q)	256	$12 \div (3 - 5 \div 2) = 24$
14	(5,5,7,J)	96	$(7 - 11 \div 5) \times 5 = 24$
15	(5,7,7,J)	96	$(5 - 11 \div 7) \times 7 = 24$

Table 2. Suggestions on target numbers and player levels.

Number of cards	Range of cards	Arithmetic operations		
		$(+, -)$	$(+, -, \times)$	$(+, -, \times, \div)$
3	A to 10	(1, 2, 3, 4, 5) (1st graders)	6 (3rd to 4th graders)	2 (3rd to 5th graders)
	A to king	Not recommend	Not recommend	Not recommend
4	A to 10	(1, 2, 3, 4, 5) (1st to 2nd graders)	6, 12 (3rd to 4th graders)	12 (4th graders and up) 24 (4th graders and up) 36 (advanced players)
	A to king	(1, 2, 3, 4, 5) (1st and 2nd graders)	12 (3rd to 4th graders)	12 (4th graders and up) 24 (4th graders and up) 36 (advanced players)

K the number 12 has the highest probability of success 0.76 and relatively high average difficulty score 5.57. In this situation, the number 6 is no longer a good choice since it has a much lower probability of success 0.69.

For the case of four-card and four-operation situation, the traditional 24-card game uses cards from A to 10 and sets the target number to 24. In this situation, the probability of success is 0.86 with average difficulty score 5.88 in the integer world and it is 0.87 with difficulty score 5.92 in the real number world. Note that when four cards are randomly selected from 40 cards (A to 10) a fraction/decimal occurs in 984 out of 91,390 ($C_{40}^{4} =$ 91390) card combinations. Among these 984 cases, there are 9 distinct card combinations. The results are listed in Table 1 (No. 1 to 9).

From the probability of success and difficulty score plots in Figure 2, it seems that the target number 24 is the largest number that has a local maximum probability of success and an average difficulty score less than 6. A score larger than 6 means more of subtraction, multiplication and division involved, which is much harder comparing with a calculation

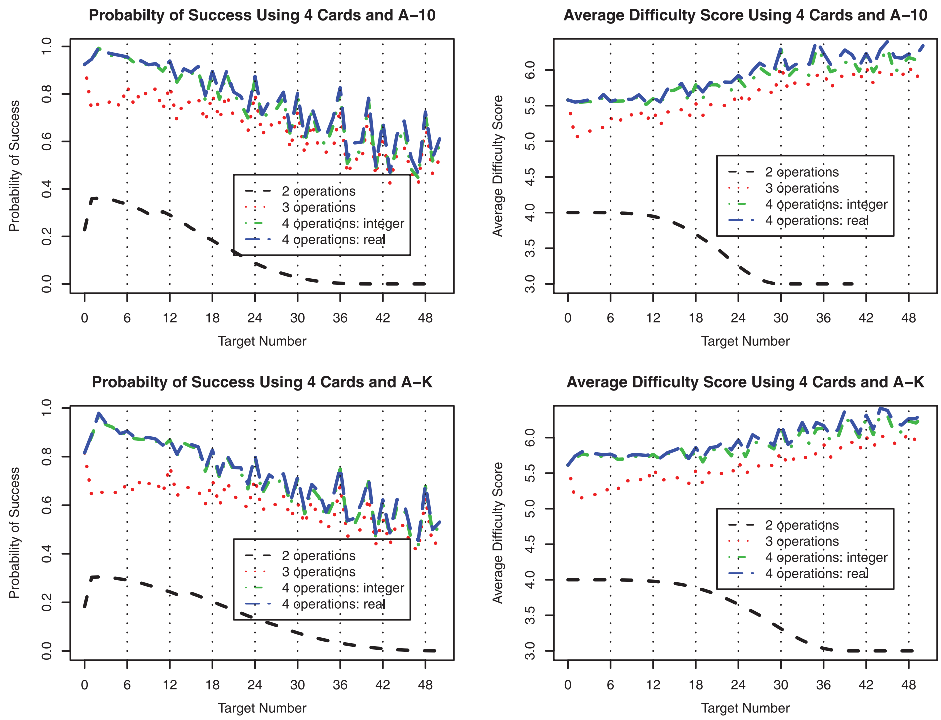

Figure 2. Probability and average difficulty score of solvable combinations using four cards randomly selected from A to 10 or from A to K.

involving more additions which tends to have a score less than 6. Therefore, considering both of solvability and difficulty, we conclude that the number 24 is a reasonable choice. Of course, if we want the game to be easier, we can choose 12, which has a higher probability of success and lower difficulty score (0.93 and 5.60 in the integer world; 0.94 and 5.61 in the real number world); if we want the game to be more challenging, the number 36 could be a good choice, which has a little lower probability of success and higher difficulty score (0.81 and 6.16 in the integer world; 0.83 and 6.24 in the real number world). From Figure 2(a) and 2(b), we conclude that it is unnecessary to try numbers larger than 40 since the probability of success drops quickly while the difficulty score stays around 6.2.

If instead 52 cards from A to K are used, for the target numbers 12, 24, and 36, the probabilities of success and average difficulty scores are (0.87, 5.76), (0.80, 5.94) and (0.75, 6.13), respectively, in the integer world and they are (0.87, 5.77), (0.80, 5.97) and (0.76, 6.17), respectively, in the real number world. For the target number 24, there are additional 6 distinct card combinations that involve decimals or fractions, which are listed in Table 1 (from No. 1 to 15).

4. Discussion

In this paper, we fully explored the probability of solvable and average difficulty score for different choices of cards and arithmetic operations. Our conclusions include the following: (1) If using three cards, we'd better choose from A to 10 to have a higher probability of success; (2) If using two arithmetic operations $(+, -)$ in the game, we suggest using multiple target numbers {1, 2, 3, 4, 5} to have a reasonably high probability (more than 0.8) of success; (3) If three arithmetic operations ($+$, $-$, $\times$) are allowed, the number 6 is recommended as the target number if using three or four cards from A to 10; the number 12 is recommended if using four cards from A to K; (4) If all four arithmetic operations ($+$, $-$, $\times$, $\div$) are allowed, the number 2 is recommended as the target number if using three cards; (5) The most complicated situation is when four cards and four arithmetic operations are used. In this situation, the numbers 12, 24 and 36 can all be the target numbers. The number 12 is the easiest and 36 the hardest. The recommendations are summarized in Table 2.

The literature on games and mathematics learning revealed that mathematical games have produced positive results regarding students' motivation, attitudes and persistence toward mathematics learning and improved student mathematics learning.[6,7–10] Further, it was suggested in [10] that when selecting mathematical games, teachers need to consider establishing clear goals and rules, have flexible learner control and choose suitable tasks at an appropriate level of challenge for students. Our current study will help teachers set up clear goals and rules, and choose appropriate levels of challenge for their students when students are engaged in playing 24-card games.

According to the Common Core State Standards for Mathematics,[3] by the end of Grade 2 students should be able to proficiently add and subtract within 20 and use the commutative and associative properties to add and subtract. Thus, it is recommended that teachers of Grades 1 and 2 can design the three- or four-card games by using addition and subtraction operations to identify the target numbers from 1 to 5. Students by the end of Grade 3 are expected to fluently multiply and divide within 100 by using strategies such as the relationship between multiplication and division, and the distributive and commutative properties. Therefore, it is appropriate that teachers can have students play the card games by adding multiplication and division operations from Grade 3 and up. This paper expands the traditional 24-card game for the purpose of mathematics education. Elementary and middle school students will develop their number sense, and enhance their knowledge in number and operation.

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