Stat 411 HW1 Fall 2023 Due August 30, 2023

0. Read §1.6, §1.7, §1.8, §1.9, §3.1, §3.2, §3.3, §3.4, §3.5, §3.6

1. Let $X_1, \ldots, X_n$ be a random sample from the Bernoulli distribution $b(1, p)$, where p is unknown. Let $Y = \sum_{i=1}^{n} X_i$.

(a) Find the distribution of Y.

(b) Show that $E(Y/n) = p$.

(c) Determine the variance of Y/n.

2. Let $\bar{X} = n^{-1} \sum_{i=1}^{n} X_i$ be the sample mean of a random sample $X_1, \ldots, X_n$ from the exponential distribution, $\mathrm{Exp}(\theta)$, with pdf $f(x) = (1/\theta)\exp\{-x/\theta\}$, $x > 0$.

(a) Show that $E(\bar{X}) = \theta$.

The following (b), (c) and (d) are for graduates only.

(b) Using the mgf technique determine the distribution of $\bar{X}$. (Hint: See §5.2.3.)

(c) Use (b) to show that $Y = 2n\bar{X}/\theta$ has a χ^2 distribution with $2n$ degrees of freedom.

(d) Based on (c), find a 95% confidence interval for θ if $n = 10$. (Hint: Find c and d such that $P(c < 2n\bar{X}/\theta < d) = 0.95$ and solve the inequalities for θ.)