Stat 521 HW4 **Fall 2012** **Due September 28, 2012**

Required Part:

0. Read §1d Convex Sets in Vector Spaces and §1e Inequalities.

1. Are these quadratic forms below positive definite? Justify your answer.

(a) $99x_1^2 - 12x_1x_2 + 48x_1x_3 + 130x_2^2 - 60x_2x_3 + 71x_3^2$

(b) $\sum_{i=1}^n x_i^2 + \sum_{1\le i<j\le n} x_ix_j$

2. Let A and B be $m \times m$ positive definite matrices such that $A - B$ is nnd. Show that $B^{-1} - A^{-1}$ is nnd too.

3. Let A be an $n \times m$ matrix of rank r, $\lambda_1, \dots, \lambda_r$ be the non-zero eigenvalues and $P_1, \dots, P_r$ be the eigenvectors of $A'A$. Show that

$$\lambda_1^{-1}P_1P_1' + \cdots + \lambda_r^{-1}P_rP_r'$$

is a g-inverse of $A'A$.

4. Let Λ be an $m \times m$ pd matrix. The inner product in $\mathbb{R}^m$ is defined by $(X, Y) = X'\Lambda Y$ and then $\|X\|^2 = (X, X)$. Let A, B be two $m \times m$ matrices. Show that

$$\|AX\| \le \|AX + BY\| \text{ for all } X, Y$$

if and only if $A'\Lambda B = 0$.

Optional Part:

5. Let A and B be $n \times m$ matrices. Show that $AA' = BB'$ if and only if $A = BP$ for some $m \times m$ matrix P satisfying $PP' = I$.