Stat 521 HW6 **Fall 2012** **Due October 19, 2012**

Required Part:

0. Read §8b Wishart Distribution and §4a Theory of Least Squares (Linear Estimation).

For Problems 1, 2, and 4, we consider a linear model $Y = X\boldsymbol{\beta} + \varepsilon$, where Y is an $n \times 1$ random vector, X is a known $n \times m$ matrix, $\boldsymbol{\beta}$ is an $m \times 1$ vector of unknown parameters, and ε is an $n \times 1$ random vector of noise with $E(\varepsilon) = 0$ and $V(\varepsilon) = \sigma^2 I_n$. All L, C, D, P, L_0 are column vectors.

1. Recall that a linear function $P'\boldsymbol{\beta}$ of $\boldsymbol{\beta}$ is called *estimable* if there exists a linear function $L'Y$ such that $E(L'Y) = P'\boldsymbol{\beta}$.

(a) Based on the definition of estimable linear function, show that all estimable linear functions of $\boldsymbol{\beta}$ form a linear space, that is,

* if $C'\boldsymbol{\beta}$ and $D'\boldsymbol{\beta}$ are estimable, then $(C + D)'\boldsymbol{\beta}$ is estimable too;
* if $C'\boldsymbol{\beta}$ is estimable and a is a scalar, then $(aC)'\boldsymbol{\beta}$ is estimable.

(b) Determine the linear space formed by all estimable linear functions of $\boldsymbol{\beta}$.

2. Consider equation $P = X'L$ where both P and L are $n \times 1$ vectors.

(a) Show that if $P = X'L$ admits a solution for L, then $L'Y$ is unbiased for $P'\boldsymbol{\beta}$.

(b) Suppose $P = X'L$ admits a solution for L. Show that there exists a unique solution $L = L_0 \in \mathcal{M}(X)$.

(c) Show that $V(L_0'Y) \leq V(L'Y)$ for any other solution L.

3. Consider the linear model $y_{ij} = \beta_0 + \beta_i + \varepsilon_{ij}$, $i = 1, 2, 3,\ j = 1, 2$ with the standard assumptions on ε, that is, ε_{ij} iid $\sim N(0, \sigma^2)$.

(a) Show that $c_1\beta_1 + c_2\beta_2 + c_3\beta_3$ is estimable if and only if $c_1 + c_2 + c_3 = 0$.

(b) Suppose the observations are $y_{11} = 0.1$, $y_{12} = -0.5$, $y_{21} = 3.2$, $y_{22} = 6.3$, $y_{31} = 4.9$, and $y_{32} = 5.9$. Find the normal equations.

(c) For the data in (b), find BLUEs for $\beta_1 - \beta_2$, $\beta_1 - \beta_3$, and $\beta_2 - \beta_3$.

Optional Part:

4. Show that in general a linear function $L'Y$ has minimum variance as an linear unbiased estimate of $E(L'Y)$ if and only if $cov(L'Y, D'Y) = 0$ for all D such that $E(D'Y) = 0$.