Stat 521 HW11 Fall 2012 Due December 5, 2012

Required Part:

0. Read §5a Minimum Variance Unbiased Estimation.

1. Suppose Y and Z are univariate random variables such that

$$Y \mid Z = z \sim N(\mu + z,\ \sigma^2), \quad Z \sim N(0, \lambda^2).$$

Find the unconditional distribution of Y.

2. Suppose $Y_1, \ldots, Y_n$ are n univariate random variables and Z is another random variable such that,

(i) Given Z, the random variables $Y_1, \ldots, Y_n$ are independent;

(ii) $Y_i \mid Z = z \sim N(\mu_i + z,\ \sigma^2)$.

Suppose $Z \sim N(0, \lambda^2)$. Find the unconditional joint distribution of $Y_1, \ldots, Y_n$.

3. Suppose an experiment produces observations $\{Y_{ij},\ i = 1, \ldots, v;\ j = 1, \ldots, b\}$ which is a collection of vb univariate random variables. Suppose $\beta_1, \ldots, \beta_b$ are b univariate random variables that are not observed.

(i) For each $j = 1, \ldots, b$, given β_j, the observations $Y_{1j}, \ldots, Y_{vj}$ are independent, and $Y_{ij} \mid \beta_j \sim N(\tau_i + \beta_j,\ \sigma^2)$.

(ii) $\beta_1, \ldots, \beta_b$ are iid $\sim N(0, \sigma_\beta^2)$.

(iii) Let $Y_j = (Y_{1j}, \ldots, Y_{vj})'$. $Y_1, \ldots, Y_b$ are independent random vectors.

(iv) $\tau_1, \ldots, \tau_v, \sigma^2 > 0, \sigma_\beta^2 > 0$ are unknown parameters (constants).

Then

(a) Find the likelihood of $\tau_1, \ldots, \tau_v, \sigma^2, \sigma_\beta^2$, based on the observations

$$\{Y_{ij}, i = 1, \ldots, v;\ j = 1, \ldots, b\}\ .$$

(b) Find the maximum likelihood estimators for $\tau_1, \ldots, \tau_v$.

(c) Find the maximum likelihood estimators for σ^2 and σ_β^2.