Stat 522 HW4 Spring 2023 Due February 6, 2023

Required Part:

1. Exercise 5.1 on pages 261.
2. Exercise 5.9 (a) (c) (e) on page 262.

   For your reference, the sample mean $\bar{x}$ is:

   ```
   95.52   164.38   55.69   93.39   17.98   31.13
   ```

   Covariance matrix **S**:

   ```
   3266.46 1343.97 731.54 1175.50 162.68 238.37
   1343.97  721.91 324.25  537.35  80.17 117.73
    731.54  324.25 179.28  281.17  39.15  56.80
   1175.50  537.35 281.17  474.98  63.73  94.85
    162.68   80.17  39.15   63.73   9.95  13.88
    238.37  117.73  56.80   94.85  13.88  21.26
   ```

3. Exercise 5.13 on page 263.

Optional Part (no need to hand in):

4. Suppose $V \sim \chi^2_n$, and $U \sim \text{Beta}(\frac{k}{2}, \frac{n-k}{2})$ with $k < n$. If U and V are independent, show that

   (i) $UV \sim \chi^2_k$, $(1-U)V \sim \chi^2_{n-k}$.

   (ii) UV and $(1-U)V$ are independent.

5. Suppose $S \sim W_n(\Sigma)$, the Wishart distribution with n degrees of freedom and $p \times p$ positive definite matrix Σ. Rewrite

   $$S = \begin{pmatrix} S_{11} & S_{12} \\ S'_{12} & S_{22} \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Sigma_{11} & 0 \\ 0 & \Sigma_{22} \end{pmatrix}$$

   where both S_{11} and Σ_{11} are $r \times r$ matrices. Show that

   $$\frac{|S|}{|S_{11}| \cdot |S_{22}|}$$

   is distributed as a product of independent beta random variables.