

BioLogic

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What is the relationship
between
logic and biology?

Classical Aspects:
Self-Reference,
Recursion,
Imaginary Values.

Symbols and
reproducibility of
symbols.

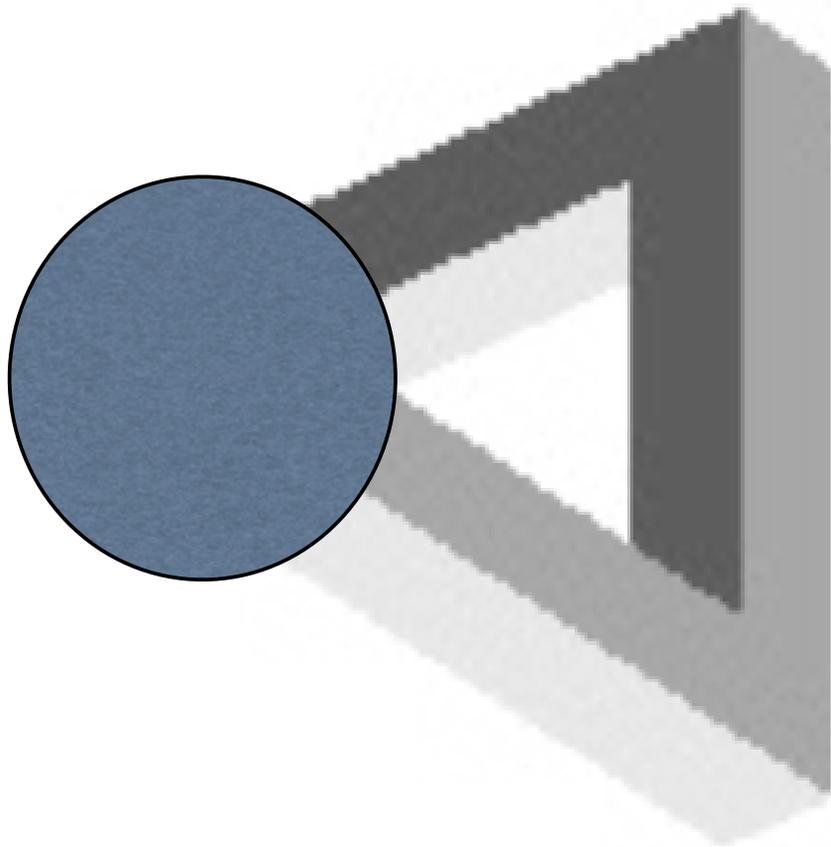
Separation of
object and reference.

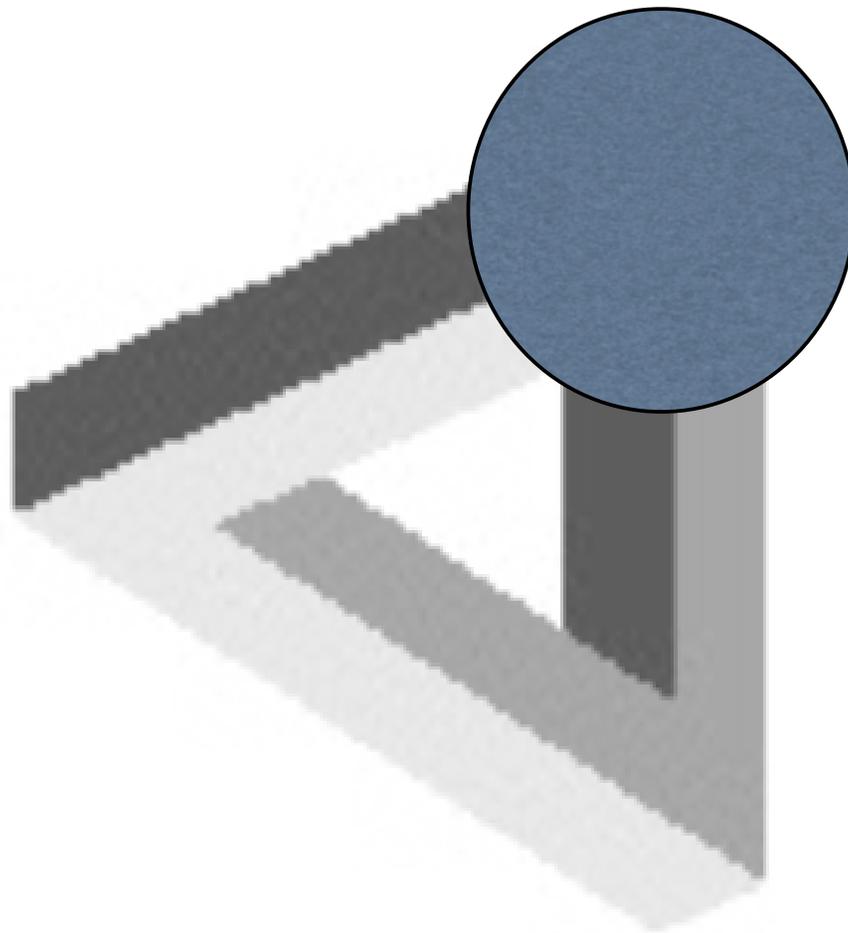


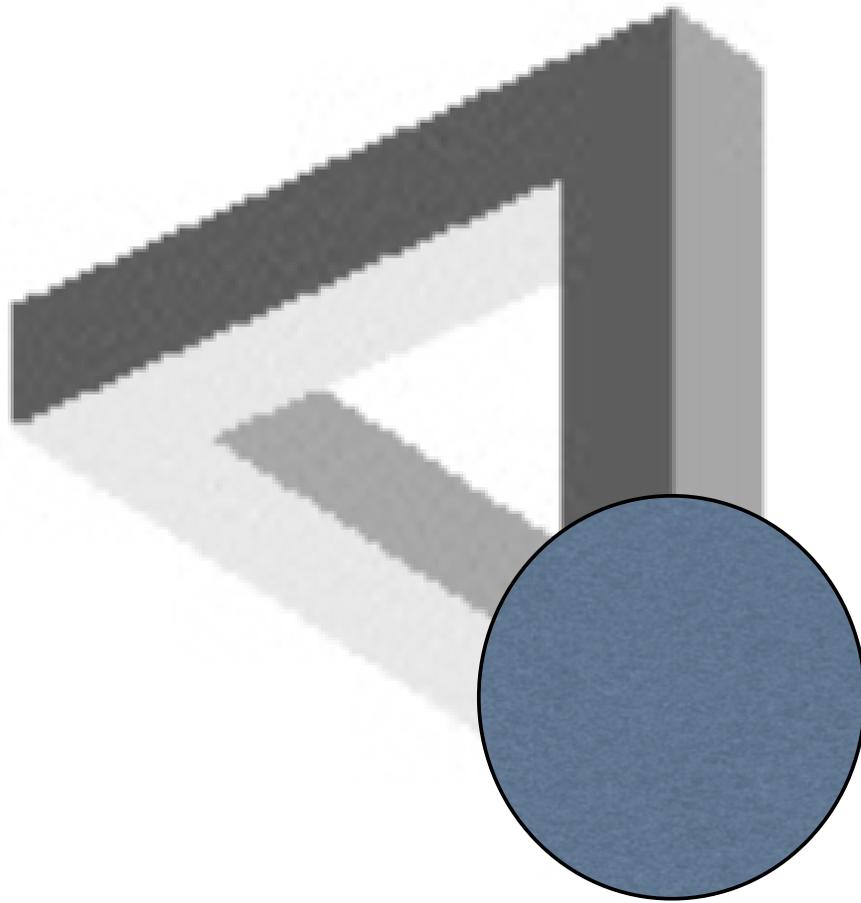


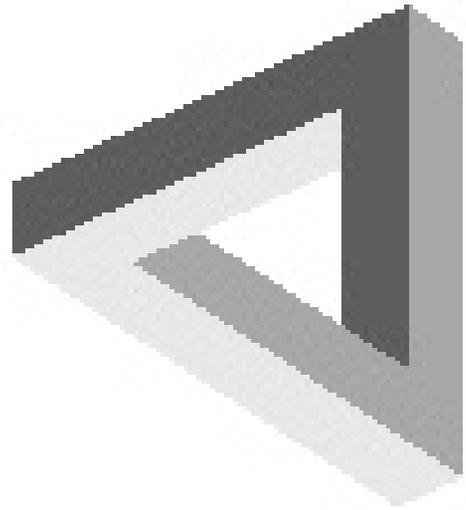
The Non-Locality of Impossibility

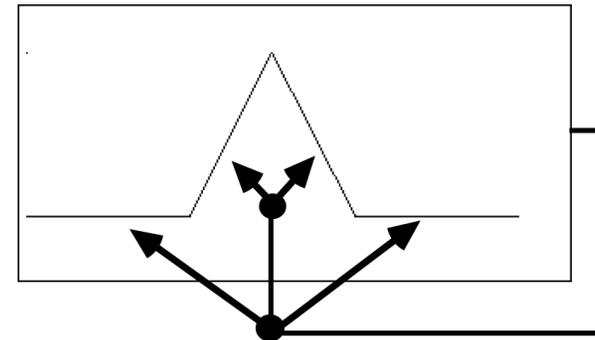
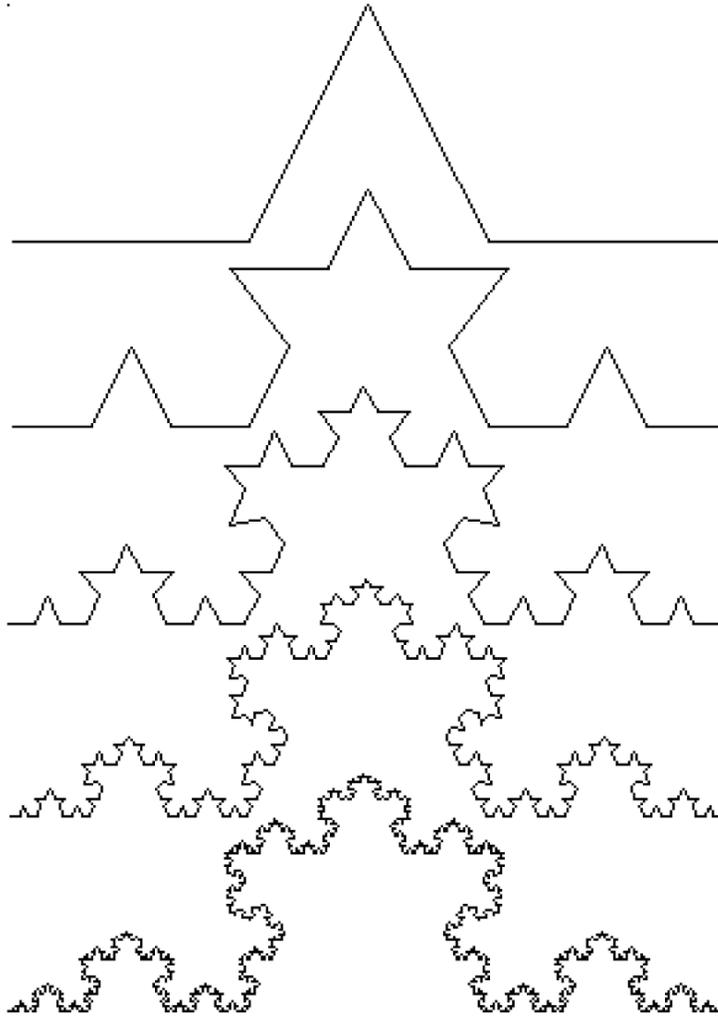












$$K = K \{ K K \} K$$

$$K = K \{ K \ K \} K$$

The Framing of
Imaginary Space.

Fixed Point and Self-Replication

$$\overline{\vee} A = \overline{AA}$$

$$\overline{\vee} \overline{\vee} = \overline{\overline{\vee} \overline{\vee}}$$

Hence

$$\overline{\vee} \overline{\vee} = \overline{\uparrow}$$

$$\overline{\square} = \overline{\dots} = \overline{\square}$$

Church-Curry Fixed Point Theorem

$$gx = F(xx)$$



$$gg = F(gg)$$

Building Machine

$$B,x \longrightarrow B,x X,x$$

(x is the blueprint for X)

Let b be the blueprint for B .

Then B,b builds itself.

$$B,b \longrightarrow B,b B,b$$

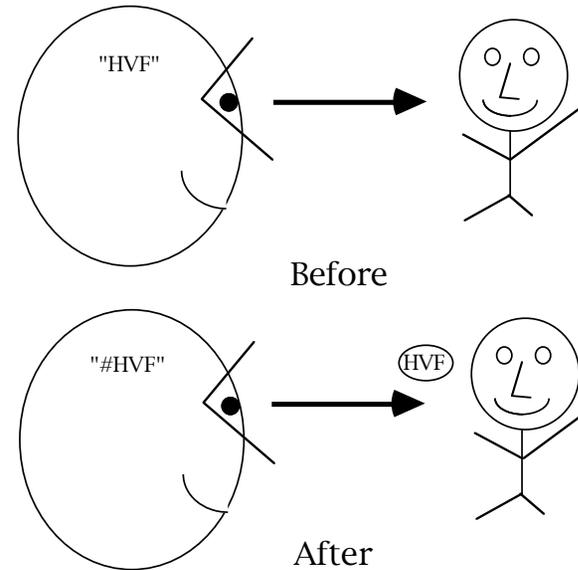
Indicative Shift

$A \longrightarrow B$

“A refers to B.”

Then

$\#A \longrightarrow BA$



Suppose that $M \longrightarrow \#$.

Then $\#M \longrightarrow \#M$. self-reference

And if $g \longrightarrow F\#$,

then $\#g \longrightarrow F\#g$.

Godelian self-reference

$$\begin{array}{ccc} M & \longrightarrow & \# \\ \# M & \longrightarrow & \# M \end{array}$$

Self Reference occurs at the Shift
of the Name M of the
Meta-Naming Operator #.

“ I am the
Observed link
Between myself
And
Observing myself.”
(Heinz von Foerster)

In a Nutshell:

Rx -----> xx

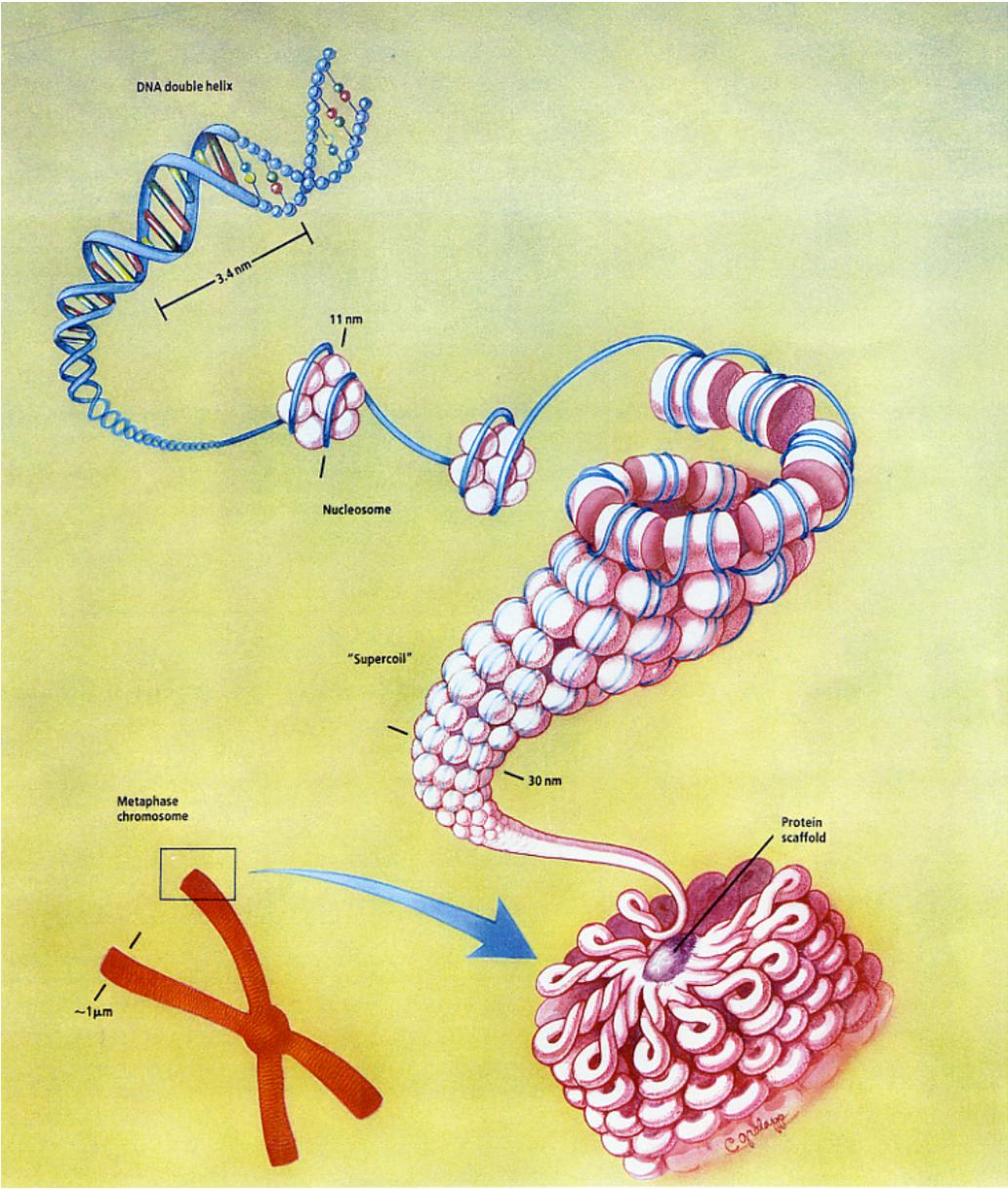
then

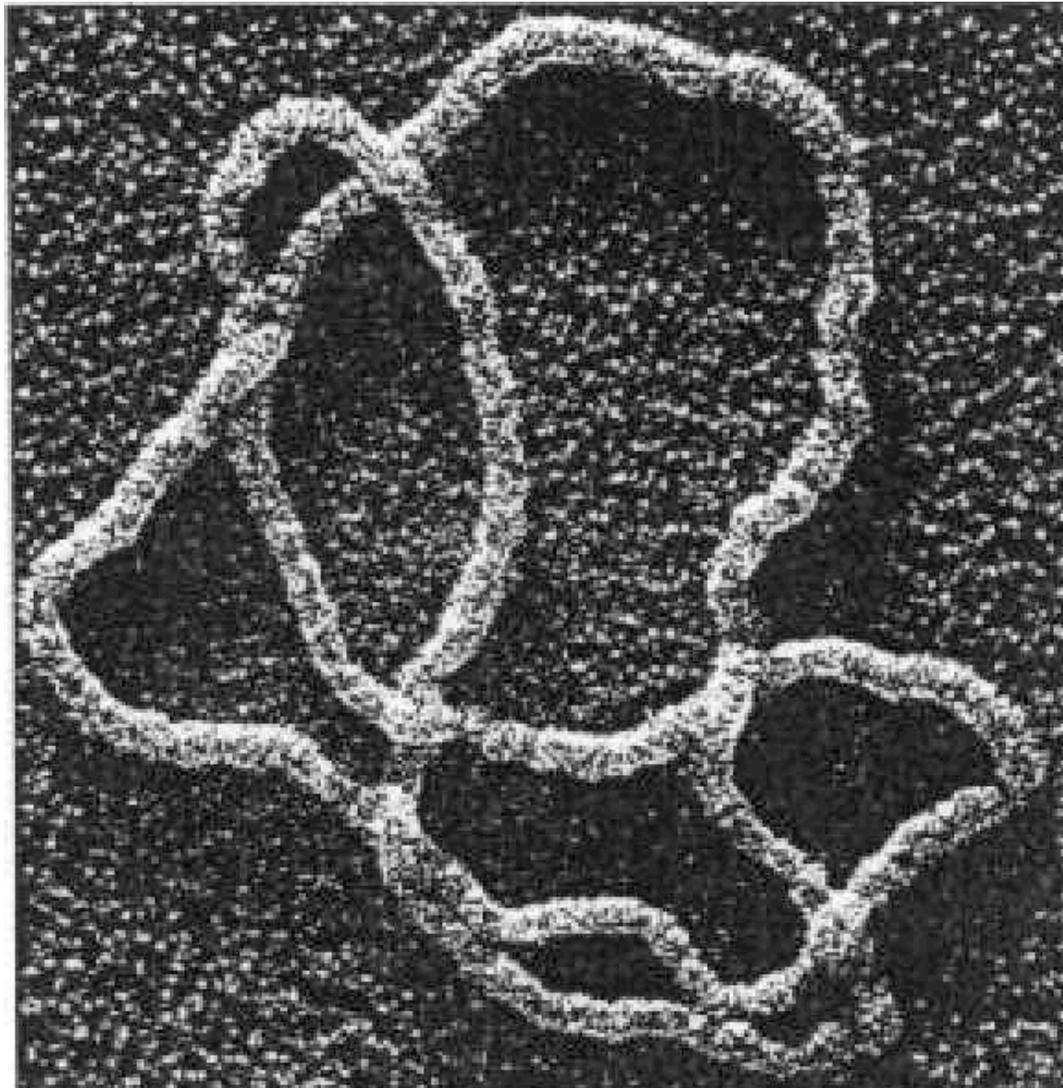
RR----->RR

So far, this is the story of the classical logic of self-replication and self-reference.

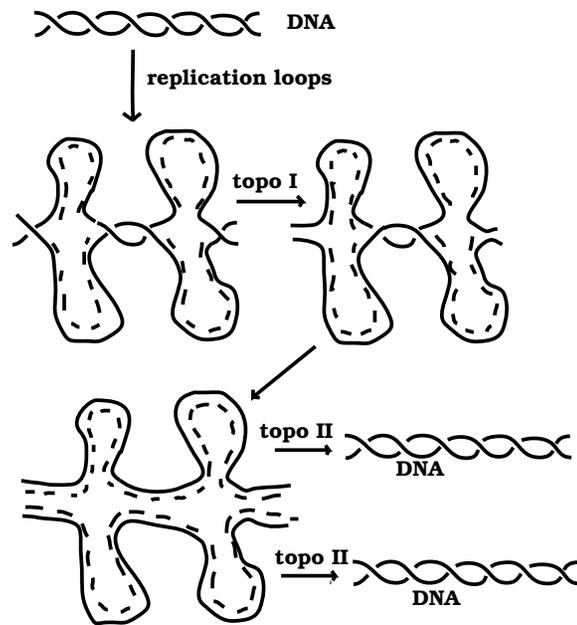
We know that DNA engages in self-replication.

How does the DNA self-rep compare with our familiar self-replication at the level of logic and recursion?





DNA is a Self-Replicating Form



$$DNA = \langle W|C \rangle$$

$$\langle W| = \langle \dots TTAGAATAGGTACGCG \dots |$$

$$|C \rangle = | \dots AATCTTATCCATGCGC \dots \rangle .$$

$$\langle W| + E \longrightarrow \langle W|C \rangle = DNA$$

$$E + |C \rangle \longrightarrow \langle W|C \rangle = DNA$$

$$\langle W|C \rangle \longrightarrow \langle W| + E + |C \rangle = \langle W|C \rangle \langle W|C \rangle$$

Self Replication Schematic

$$DNA = \langle \text{Watson} | \text{Crick} \rangle$$

E = Environment

The base pairs are *AT* (Adenine and Thymine) and *GC* (Guanine and Cytosine). Thus if

$$\langle W| = \langle \dots TTAGAATAGGTACGCG \dots |$$

Then

$$|C \rangle = | \dots AATCTTATCCATGCGC \dots \rangle .$$

Symbolically we can oversimplify the whole process as

$$\langle W| + E \longrightarrow \langle W|C \rangle = DNA$$

$$E + |C \rangle \longrightarrow \langle W|C \rangle = DNA$$

$$\langle W|C \rangle \longrightarrow \langle W| + E + |C \rangle = \langle W|C \rangle \langle W|C \rangle$$

This is the formalism of DNA replication.

Where is the repetition in the DNA self-replication? The repetition and the replication are no longer separated. The repetition occurs not syntactically, but directly at the point of replication. Note the device of pairing or mirror imaging. A calls up the appearance of T and G calls up the appearance of C . $\langle W|$ calls up the appearance of $|C \rangle$ and $|C \rangle$ calls up the appearance of $\langle W|$. Each object O calls up the appearance of its *dual or paired object* O^* . O calls up O^* and O^* calls up O . The object that replicates is implicitly a repetition in the form of a pairing of object and dual object.

The repetition is inherent in the replicand
in the sense that the dual of a form
is inherent in the form.

OO^* replicates via

$$O \longrightarrow OO^*$$

$$O^* \longrightarrow OO^*$$

whence

$$OO^* \longrightarrow O \quad O^* \longrightarrow OO^* \quad OO^*.$$

DNA = <>

DNA = <> \longrightarrow < *E* > \longrightarrow <><> = DNA DNA.

E is the “environment”.

E is replaced by ><.

If <> is a container,
then >< is an extainer.

<><> = < >< >

>< >< = ><><

Quantum Formalism

Dirac [5] introduced the “bra -(c)-ket” notation $\langle A|B \rangle = A^*B$ for the inner product of complex vectors $A, B \in H$. He also separated the parts of the bracket into the *bra* $\langle A|$ and the *ket* $|B \rangle$. Thus

$$\langle A|B \rangle = \langle A| |B \rangle$$

Dirac can write the “ket-bra” $|A \rangle \langle B| = AB^*$.

$$P = |A \rangle \langle B|$$

$$P^2 = |A \rangle \langle B| |A \rangle \langle B| = |A \rangle \langle B| A \rangle \langle B|$$

$$= \langle B| A \rangle |A \rangle \langle B| = \langle B| A \rangle P.$$

Sum over Paths (Possibilities)

If $\{|C_1\rangle, |C_2\rangle, \dots, |C_n\rangle\}$ is an orthonormal basis for H , and $P_i = |C_i\rangle\langle C_i|$, then for any vector $|A\rangle$ we have

$$|A\rangle = \langle C_1|A\rangle |C_1\rangle + \dots + \langle C_n|A\rangle |C_n\rangle .$$

Hence

$$\langle B|A\rangle = \langle C_1|A\rangle \langle B|C_1\rangle + \dots + \langle C_n|A\rangle \langle B|C_n\rangle$$

$$= \langle B|C_1\rangle \langle C_1|A\rangle + \dots + \langle B|C_n\rangle \langle C_n|A\rangle$$

$$= \langle B| [|C_1\rangle\langle C_1| + \dots + |C_n\rangle\langle C_n|] |A\rangle$$

$$= \langle B|1|A\rangle .$$

$$\sum_{k=1}^n P_k = \sum_{k=1}^n |C_k\rangle\langle C_k| = 1$$

In the quantum context one may wish to consider the probability of starting in state $|A\rangle$ and ending in state $|B\rangle$. The square of the probability for this event is equal to $|\langle B|A\rangle|^2$. This can be refined if we have more knowledge. If it is known that one can go from A to C_i ($i = 1, \dots, n$) and from C_i to B and that the intermediate states $|C_i\rangle$ are a complete set of orthonormal alternatives then we can assume that $\langle C_i|C_i\rangle = 1$ for each i and that $\sum_i |C_i\rangle\langle C_i| = 1$. This identity now corresponds to the fact that 1 is the sum of the probabilities of an arbitrary state being projected into one of these intermediate states.

Quantum Formalism and DNA Replication

We compare

$$E = |C \rangle \langle W|$$

and

$$1 = \sum_k |C_k \rangle \langle C_k|.$$

That the unit 1 can be written as a sum over the intermediate states is an expression of how the environment (in the sense of the space of possibilities) impinges on the quantum amplitude, just as the expression of the environment as a soup of bases ready to be paired (a classical space of possibilities) serves as a description of the biological environment. The symbol $E = |C \rangle \langle W|$ indicated the availability of the bases from the environment to form the complementary pairs. The projection operators $|C_i \rangle \langle C_i|$ are the possibilities for interlock of initial and final state through an intermediate possibility. In the quantum mechanics the special pairing is not of bases but of a state and a possible intermediate from a basis of states. It is through this common theme of pairing that the conceptual notation of the bras and kets lets us see a correspondence between such separate domains.

Quantum Copies are not Possible

Proof of the No Cloning Theorem. In order to have a quantum process make a copy of a quantum state we need a unitary mapping $U : H \otimes H \rightarrow H \otimes H$ where H is a complex vector space such that there is a fixed state $|X\rangle \in H$ with the property that

$$U(|X\rangle |A\rangle) = |A\rangle |A\rangle$$

for any state $|A\rangle \in H$. ($|A\rangle |B\rangle$ denotes the tensor product $|A\rangle \otimes |B\rangle$.)
Let

$$T(|A\rangle) = U(|X\rangle |A\rangle) = |A\rangle |A\rangle.$$

Note that T is a linear function of $|A\rangle$. Thus we have

$$T|0\rangle = |0\rangle |0\rangle = |00\rangle,$$

$$T|1\rangle = |1\rangle |1\rangle = |11\rangle,$$

$$T(\alpha|0\rangle + \beta|1\rangle) = (\alpha|0\rangle + \beta|1\rangle)(\alpha|0\rangle + \beta|1\rangle).$$

But

$$T(\alpha|0\rangle + \beta|1\rangle) = \alpha|00\rangle + \beta|11\rangle.$$

Hence

$$\begin{aligned} \alpha|00\rangle + \beta|11\rangle &= (\alpha|0\rangle + \beta|1\rangle)(\alpha|0\rangle + \beta|1\rangle) \\ &= \alpha^2|00\rangle + \beta^2|11\rangle + \alpha\beta|01\rangle + \beta\alpha|10\rangle \end{aligned}$$

From this it follows that $\alpha\beta = 0$. Since α and β are arbitrary complex numbers, this is a contradiction. \square

The proof of the no-cloning theorem depends crucially on the linear superposition of quantum states and the linearity of quantum process. By the time we reach the molecular level and attain the possibility of copying DNA molecules we are copying in a quite different sense than the ideal quantum copy that does not exist. The DNA and its copy are each quantum states, but they are different quantum states! That we see the two DNA molecules as identical is a function of how we filter our observations of complex and entangled quantum states. Nevertheless, the identity of two DNA copies is certainly at a deeper level than the identity of the two letters “i” in the word identity. The latter is conventional and symbolic. The former is a matter of physics and biochemistry.

On the Mathematical Side

{ }

Each left or right bracket in itself makes a distinction. The two brackets are distinct from one another by mirror imaging, which we take to be a notational reflection of a fundamental process (of distinction) whereby two forms are identical (indistinguishable) except by comparison in the space of an observer. The observer *is* the distinction between the mirror images. Mirrored pairs of individual brackets interact to form either a *container*

$$C = \{ \}$$

or an *extainer*

$$E = \} \{.$$

These new forms combine to make:

$$CC = \{\}\{\} = \{E\}$$

and

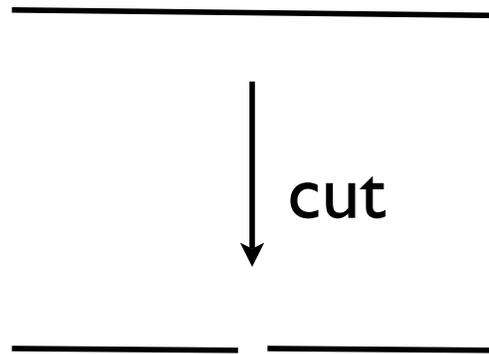
$$EE = \}\{\}\{=\}C\{.$$

$$EE = \}\{\}\{=\}C\{= C\}\{= CE.$$

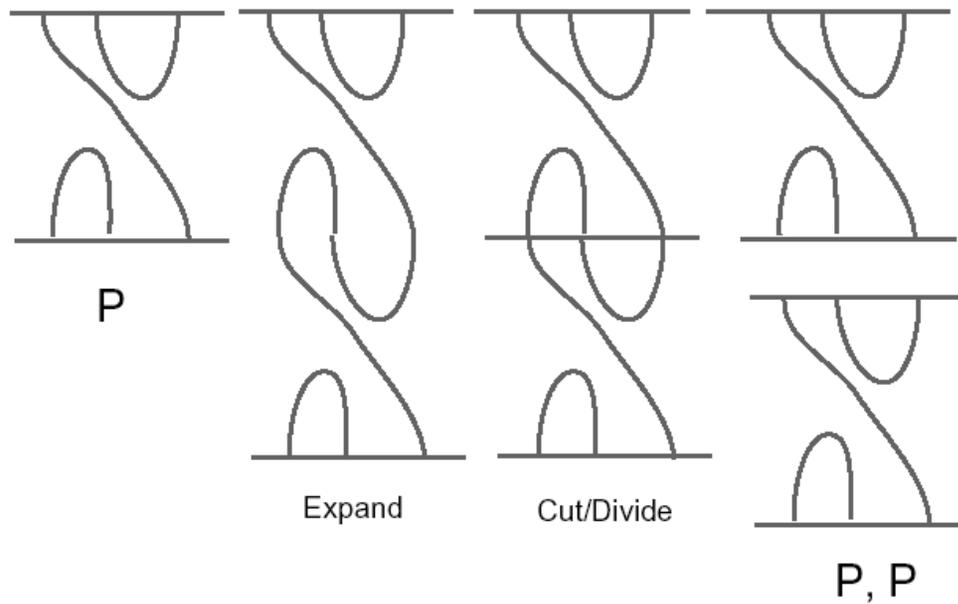
**It is natural to make the container
the analog of a scalar quantity
and make it commute with
individual brackets.**

We can also regard $EE = \{ \} E$ as symbolic of the emergence of DNA from the chemical substrate. Just as the formalism for reproduction ignores the topology, this formalism for emergence ignores the formation of the DNA backbone along which are strung the complementary base pairs. In the biological domain we are aware of levels of ignored structure.

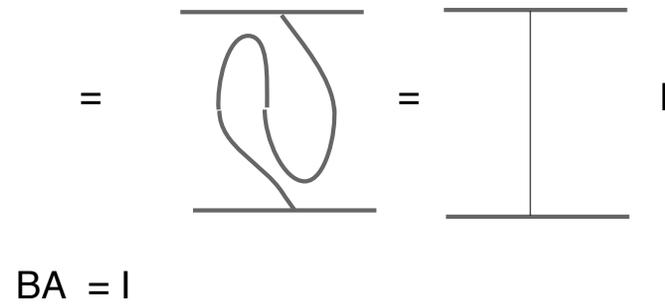
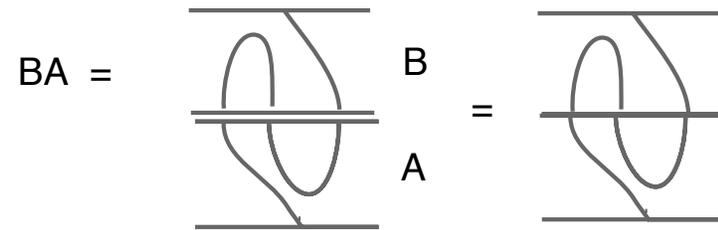
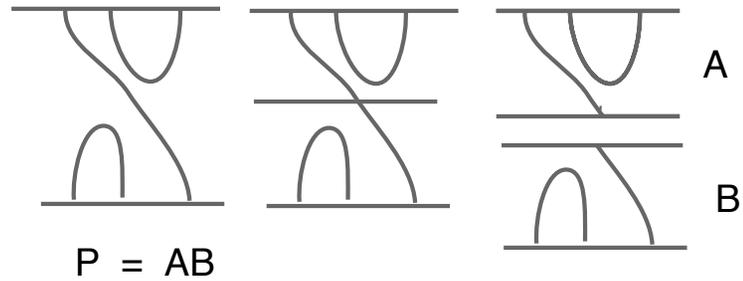
Simplest Replication



Topological Replication



Why the topological self-rep worked.



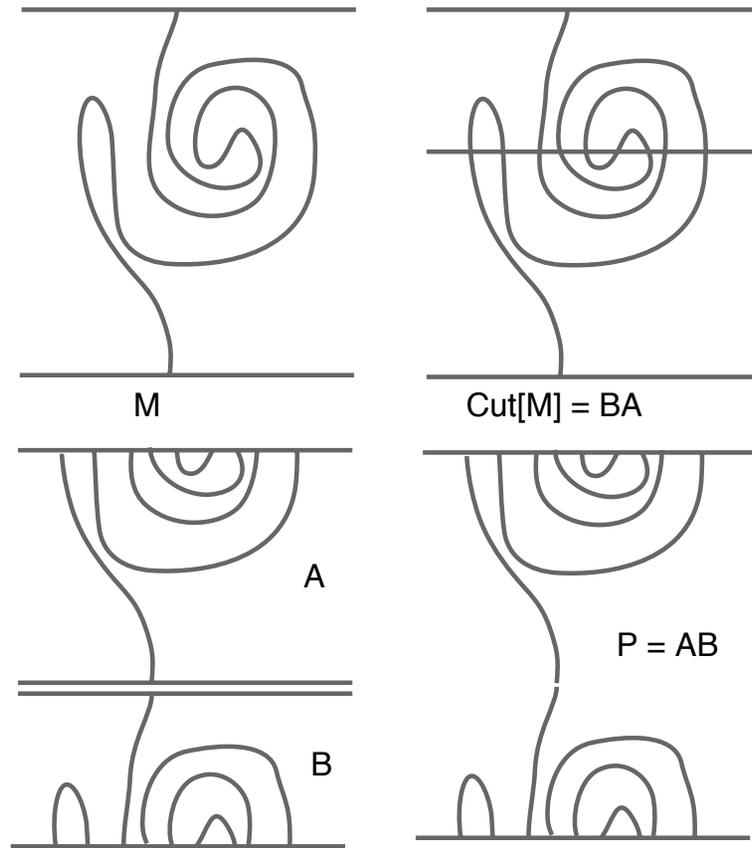


Figure 2.4 - Constructing a new P with $PP = P$.

The Algebraic Realm

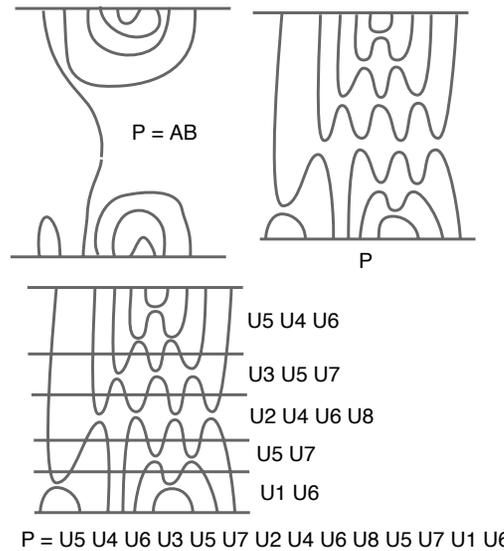


Figure 2.5 - Writing P as a product of standard generators.

The *Temperley Lieb algebra* TL_n is an algebra over a commutative ring k with generators $\{1, U_1, U_2, \dots, U_{n-1}\}$ and relations

$$U_i^2 = \delta U_i,$$

$$U_i U_{i\pm 1} U_i = U_i,$$

$$U_i U_j = U_j U_i, |i - j| > 1,$$

Protein Folding

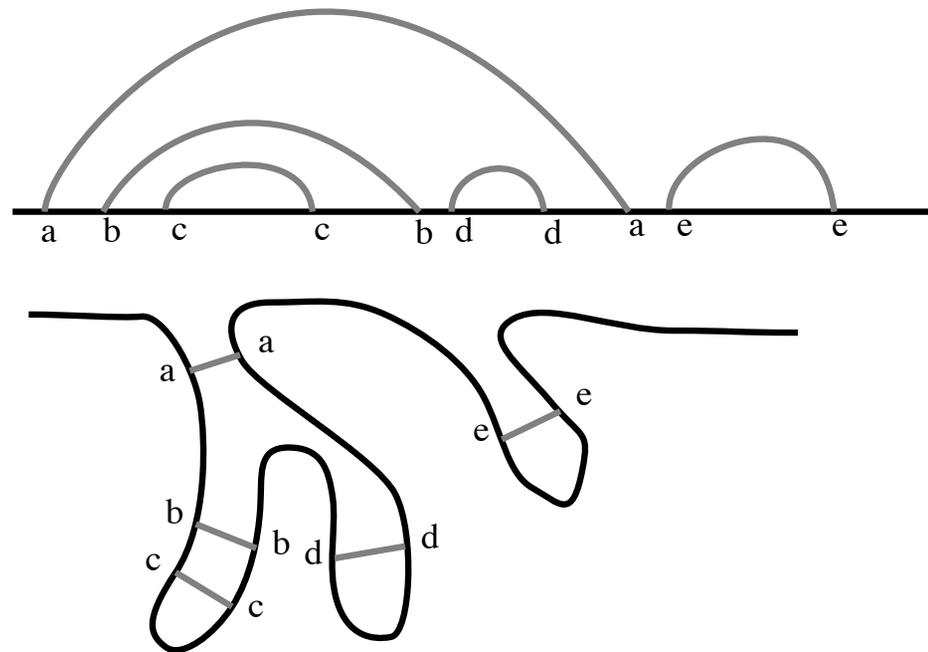


Figure 3 - Secondary Structure

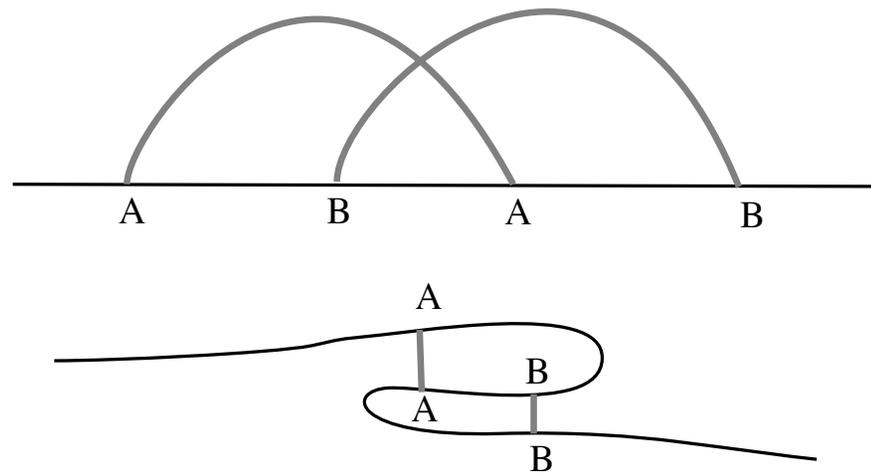


Figure 4 - A Tertiary Structure - $\langle a | \langle b | | a \rangle | b \rangle$

Cell Self-Assembly Arising From a Substrate of Rules and Interactions

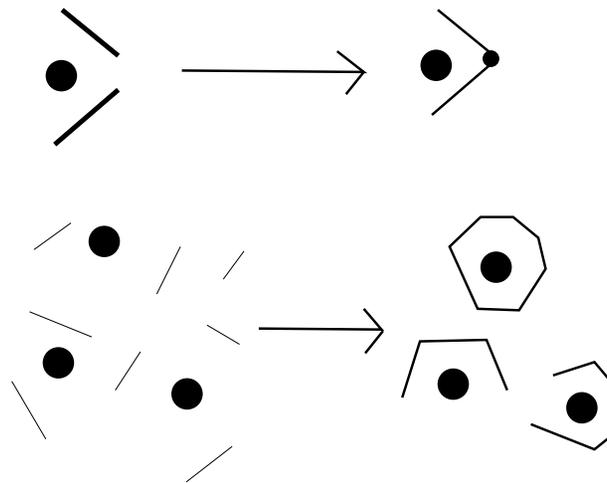


Figure 5 - Proto-Cells of Maturana, Uribe and Varela

In the course of time the catalysts (basically separate from one another due to lack of bonding) become surrounded by circular forms of bonded or partially bonded substrate. A distinction (in the eyes of the observer) between inside (near the catalyst) and outside (far from a given catalyst) has spontaneously arisen through the “chemical rules”. Each catalyst has become surrounded by a proto-cell. No higher organism has formed here, but there is a hint of the possibility of higher levels of organization arising from a simple set of rules of interaction. *The system is not programmed to make the proto-cells.* They arise spontaneously in the evolution of the structure over time.

Could the Glider Gun Arise Spontaneously?

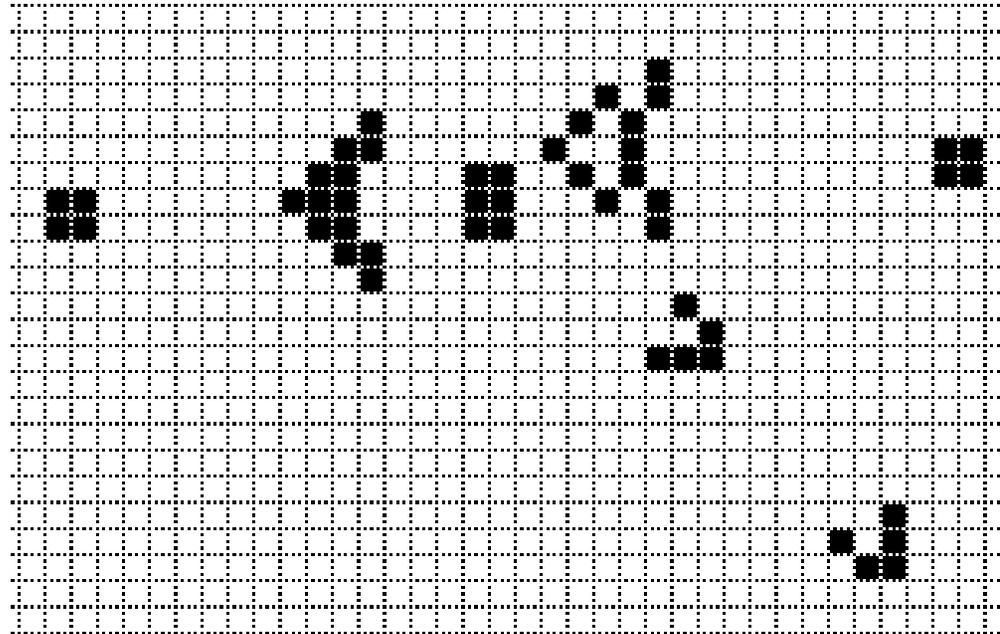


Figure 6 - Glider Gun and Gliders

Other Examples

Topological Processes

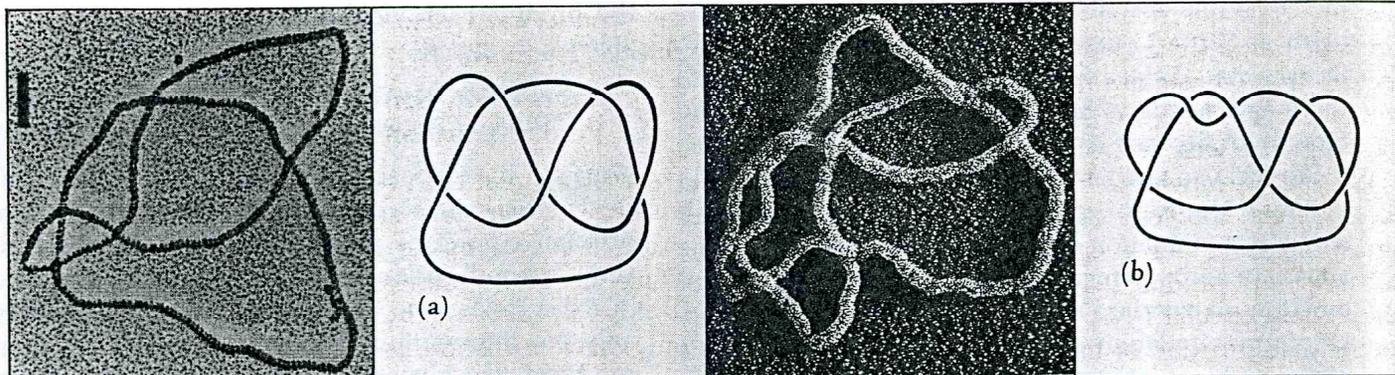
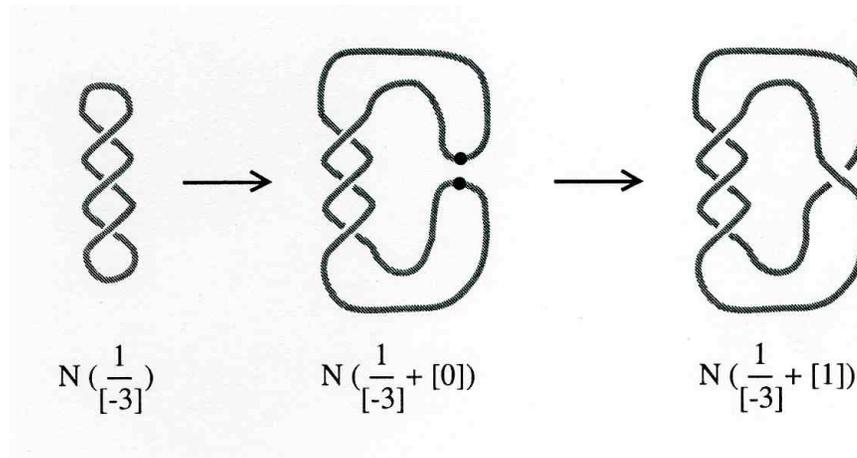


Figure 8. DNA 4-plats ($Tn3$) (a) shows the Whitehead link $[1,1,1,1,1]$; (b) the knot $6_2 [1,2,1,1,1]$.

DNA Recombination



Tangle Model: Ernst & Sumners, 1989

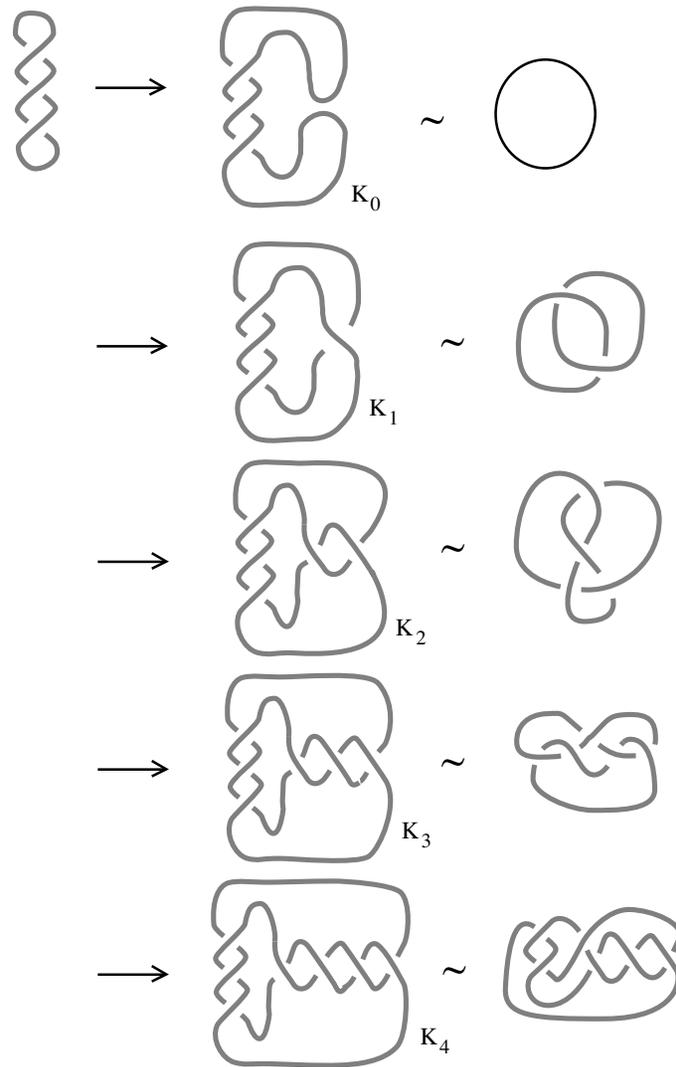


Figure 28 - Processive Recombination with $S = [-1/3]$.

Summary:

In this paper we have covered a wide ground of ideas related to the foundations of mathematics and its relationship with biology and with physics. There is much more to explore in these domains. The result of our exploration has been the articulation of a mathematical region that lies in the crack between set theory and its notational foundations. We have articulated the concepts of container $\langle \rangle$ and extainer $\rangle \langle$ and shown how the formal algebras generated by these forms encompass significant parts of the logic of DNA replication, the Dirac formalism for quantum mechanics, formalism for protein folding and the Temperley Lieb algebra at the foundations of topological invariants of knots and links. It is the mathematician's duty to point out formal domains that apply to a multiplicity of contexts. In this case we suggest that it is just possible that there are deeper connections among these apparently diverse contexts that are only hinted at in the steps taken so far. The common formalism can act as compass and guide for further exploration.