

Model Theory 1: Homework 3

Due on Friday October 13th, 2006

1. Let T be an \mathcal{L} -theory. In the lectures we defined T^\forall and showed that if $\mathcal{M} \models T$ and $\mathcal{A} \subseteq \mathcal{M}$ is a substructure then $\mathcal{A} \models T^\forall$.
Give an appropriate definition of T^\exists and show that if $\mathcal{M} \models T$ and $\mathcal{M} \subseteq \mathcal{A}$ is a superstructure then $\mathcal{A} \models T^\exists$.
2. Give two non-isomorphic models of DLO of cardinality 2^{\aleph_0} , and thus show that DLO is not 2^{\aleph_0} -categorical. Can you extend your idea to any other uncountable cardinals?
3. Consider $\langle \mathbb{Q}; < \rangle$.
 - a) Which subsets of \mathbb{Q}^1 and \mathbb{Q}^2 are definable without parameters?
 - b) Which subsets of \mathbb{Q}^1 are definable with parameters?