Model Theory 1: Homework 4
(revised)

Due on Wednesday November 1st, 2006

Let F be an infinite field, let $\mathcal{L} = \{0, +, (s_\lambda)_{\lambda \in F}\}$ where each s_λ is a unary function symbol, and let T_F be the theory of infinite F-vector spaces in this language, with s_λ interpreted as scalar multiplication by λ. We have previously seen that T_F is categorical in all cardinals greater than $\operatorname{card} F$.

Let V be an F-vector space and A a subset of V.

1. Show that T_F has quantifier elimination in $\mathcal{L}$.

2. Give a description in terms of linear algebra of the automorphisms of V which fix A pointwise. The group of all of these automorphisms is written $\operatorname{Aut}(V/A)$. What are the orbits of this automorphism group on V and V^2?

3. Give a description of all the types in $S_1^V(\emptyset)$ and $S_2^V(\emptyset)$. Which of these types are isolated?

4. If $A \neq \emptyset$, give a description of all the complete types in $S_1^V(A)$. How many of them are there? Which of them are isolated?

5. Given a subset A of V, find a model V' containing A such that every n-type over A realized in V' is isolated. (That is, V' omits every nonisolated type.)

6. A theory T is *strongly minimal* iff for every model $\mathcal{M}$ of T, the only subsets of M^1 which are definable with parameters are finite or cofinite. Show that T_F is strongly minimal.