**Model Theory 1: Homework 5**

Due on Friday November 17th, 2006

1. Let $F$ be an infinite field of cardinality $\lambda$. What are the $\kappa$-saturated models of the theory of $F$-vector spaces?

2. Let $F_3$ be the field with three elements, and let $T$ be the theory of infinite $F_3$-vector spaces. Show that $T$ is complete and *totally categorical*, that is, categorical in all infinite cardinals. What is different about the type of a linearly-independent $n$-tuple in this example and in the example of vector spaces over an infinite field? If $A$ is a set of parameters, how large is $S_1^V(A)$?

3. Let $\mathcal{M} \models DLO$ and $A \subseteq \mathcal{M}$. Put an upper bound on the cardinality of $S_1^{\mathcal{M}}(A)$ in terms of the cardinality of $A$. For $\mathcal{M} = \mathbb{Q}$ and $|A| = \aleph_0$, show by examples that this upper bound can be reached but is not always reached.

4. We have seen that isolated types are useful. However, there are theories with no isolated types. Do exercise 4.5.16, parts a) — h), of Marker's book, which give a construction of such a theory.

5. Show that if $\mathcal{M}$ and $\mathcal{N}$ have the same theory and cardinality and are both homogeneous and realize the same $n$-types for each $n \in \mathbb{N}$ then $\mathcal{M} \cong \mathcal{N}$.
   [This is theorem 4.3.23 of Marker's book. I suggest you try to prove it without the book first, and then look at the proof there if you do not have enough ideas yourself.]

6. Show that atomic models are $\aleph_0$-homogeneous. Use this and the above question to prove that prime models of complete theories in a countable language are unique up to isomorphism.

7. [Not to be handed in, but worth doing] Write out a complete proof of theorem 4.2.4, which says we can omit a countable family of nonisolated types.