Some terminology for functions

Terminology for functions seems to vary; in particular the word *range* is used for different things. There are also two different definitions of a function: one in which the codomain matters and one in which it does not. Things are even less clear when it comes to partial functions. These are my conventions, which I encourage other people to use.

A *function* f is defined by three pieces of data: a *domain*, a *codomain*, and a *graph*. The domain and codomain are sets, say X and Y respectively, and the graph is a subset G of $X \times Y$. The graph must satisfy: for every $x \in X$ there is exactly one $y \in Y$ such that $(x, y) \in G$. If $(x, y) \in G$ we write $f(x) = y$.

I write $X = \mathrm{dom}(f)$, $Y = \mathrm{cod}(f)$ and $G = \mathrm{graph}(f)$.

The *image* of f is $\mathrm{im}\, f = \{f(x) \mid x \in \mathrm{dom}(f)\}$, a subset of the codomain. For purists, the definition is

$$\mathrm{im}(f) = \{y \in \mathrm{cod}(f) \mid \exists x \in \mathrm{dom}(f)[y = f(x)]\}\,.$$

A *partial function* is defined as before, except that the graph satisfies a weaker property: for every $x \in X$ there is at most one $y \in Y$ such that $(x, y) \in G$.

The terminology of domain, codomain, graph, and image is as before. In addition we define the *preimage* of f to be the subset of the domain:

$$\mathrm{preim}(f) = \{x \in \mathrm{dom}(f) \mid \exists y \in \mathrm{cod}(f)[y = f(x)]\}\,.$$

I define two functions to be equal iff their domains, codomains and graphs are equal. (For total functions the domain is recoverable from the graph of course, but there seems to be no advantage in leaving it out of the definition and several advantages of keeping it.)

I avoid using the word “range” which is used to mean both codomain and image. For partial functions I avoid using the word domain to mean preimage. It is usually important to distinguish the two.