CS / MCS 401 Week #5–6 Exercises (Fall 2007)

H. (postponed from Week #4 exercises) Consider the recurrence

$$C(n) = C(0.8n) + C(0.5n) + C(0.2n) + n, \quad C(1) = 1,$$

ignoring the fact that $0.8n$, $0.5n$, and $0.2n$ may not be integers. If the recurrence has a solution of the form

$$C(n) = a\,n^b + c\,n + d$$

for some real numbers a, b, c, and d, what must the value of b be? Compute your answer to two decimal places.

I. How many inversions are there in the array

a = (41 16 74 33 66 54)?

How many comparisons would straight insertion sort perform in sorting this array? How many exchanges would it perform?

J. Decide whether each strict partial order $\prec$ on the set S is a strict weak order. Recall that a strict weak order is a strict partial order in which the relation ~ defined by

$$a \sim b \text{ if } \textbf{not}(a \prec b) \textbf{ and not}(b \prec a)$$

is an equivalence relation.[1] If $\prec$ is a strict weak order, give a simple, intuitive description of the equivalence classes of the equivalence relation ~ defined above. (You need not prove that $\prec$ is a strict weak order.) If $\prec$ is a not strict weak order, give a counterexample.

i) S = X–Y plane, $(x,y) \prec (u,v)$ if $x < u$ and $y < v$.

ii) S = X–Y plane, $(x,y) \prec (u,v)$ if $x^2 + 4y^2 < u^2 + 4v^2$.

iii) S = X–Y plane, $(x,y) \prec (u,v)$ if $x - y < u - v$.

iv) S = (X–Y plane), $(x,y) \prec (u,v)$ if $x < u - 1$.

v) S = real numbers, $a \prec b$ if $\lfloor a \rfloor < \lfloor b \rfloor$.

vi) S = real numbers, $a \prec b$ if $a - \lfloor a \rfloor < b - \lfloor b \rfloor$.

vii) S = positive integers, $a \prec b$ if a is a proper divisor of b (i.e., b/a is an integer different from 1).

Exercise 6.1-6

Exercises 6.3-1 and 6.3-2

Exercise 6.4-1 (As in Fig 6.4, start with the heap just after is has been constructed by *build-max-heap*(). You may limit yourself to illustrating the first three steps. One step consists of removing the root from the heap and then restoring the remaining elements to a heap.)

Exercise 6.5.7

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[1] To demonstrate that ~ is an equivalence relationship, it is sufficient to show that ~ transitive. Reflexivity and symmetry automatically hold for this particular relation.