Problem on FFT from Spring 2007 Midterm

5. (4 points) Let $A(x) = 1 + 3x - 2x^2 + 4x^3 + 5x^4 - 3x^5 + 2x^6 - x^7$, a polynomial of degree bound 8. We want to evaluate $A(x)$ at certain eighth roots of unity, employing the technique used in the FFT. (If $\omega_8 = e^{2\pi i/8} = (1+i)/\text{sqrt}(2)$, the eighth roots of unity are ω_8^k, $k = 0, 1, ..., 7$.)

We define polynomials

$$A_E(x) = 1 - 2x + 5x^2 + 2x^3 \quad \textit{and} \quad A_O(x) = 3 + 4x - 3x^2 - x^3$$

and compute the values of $A_E(x)$ and $A_O(x)$ at the fourth roots of unity (1, ω_4, ω_4^2, ω_4^3 where $\omega_4 = e^{2\pi i/4} = i$, that is, 1, i, -1, and $-i$). The values are

Polynomial *P*(x)	***P*(1)**	***P*(ω_4)**	***P*(ω_4^2)**	***P*(ω_4^3)**
$A_E(x)$	6	$-4-4i$	6	$-4+4i$
$A_O(x)$	3	$6+5i$	-3	$6-5i$

Using the information in the table above, calculate the values of $A(\omega_8)$ and $A(\omega_8^5)$. Show how you obtain your answer. Each answer should be in the form $\alpha + \beta i$ for some real numbers α and β.