Exercise set #6, exercises K and L— Summer 2007

K. The dynamic programming algorithm for finding the cost of computing $M_1 \times M_2 \times ... \times M_{18}$ using optimal order gave the following output:

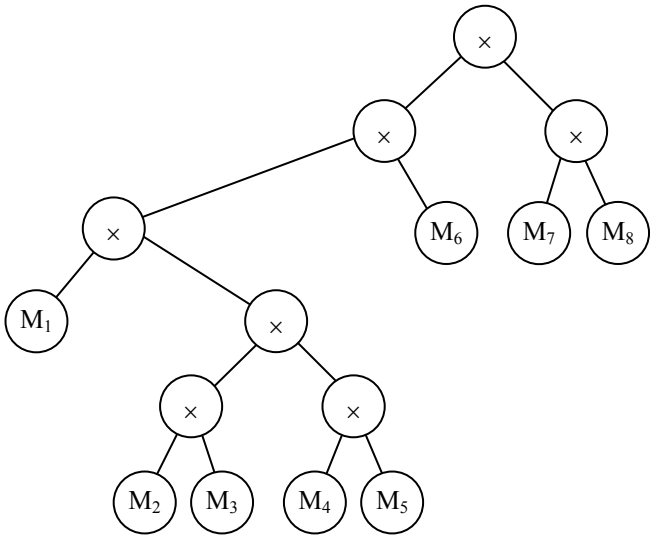
i	j	m_{ij}	v_{ij}
1	2	150	1
2	3	360	2
3	4	180	3
4	5	240	4
5	6	120	5
6	7	216	6
7	8	108	7
8	9	72	8
9	10	80	9
10	11	600	10
11	12	1200	11
12	13	360	12
13	14	168	13
14	15	231	14
15	16	3465	15
16	17	990	16
17	18	1080	17
1	3	330	2
2	4	330	2
3	5	240	4
4	6	480	4
5	7	390	6
6	8	156	6
7	9	156	8
8	10	260	8
9	11	380	10
10	12	1080	11
11	13	810	11
12	14	675	13
13	15	495	13
14	16	1716	15
15	17	1144	15
16	18	1254	17
1	4	405	2
2	5	360	2
3	6	312	5
4	7	888	5
5	8	196	5
6	9	188	8

i	j	m_{ij}	v_{ij}
7	10	308	8
8	11	650	8
9	12	620	11
10	13	930	10
11	14	1020	13
12	15	1086	13
13	16	2796	13
14	17	1186	14
15	18	1312	17
1	5	450	2
2	6	492	2
3	7	474	6
4	8	316	4
5	9	236	8
6	10	316	8
7	11	668	8
8	12	764	8
9	13	668	12
10	14	1014	13
11	15	1371	13
12	16	4101	13
13	17	1234	13
14	18	1258	17
1	6	552	2
2	7	744	2
3	8	388	3
4	9	412	8
5	10	376	8
6	11	656	8
7	12	824	8
8	13	722	8
9	14	710	13
10	15	1293	13
11	16	3876	13
12	17	1474	12
13	18	1426	17
1	7	759	2
2	8	448	2
3	9	412	8

i	j	m_{ij}	v_{ij}
4	10	636	8
5	11	726	8
6	12	840	8
7	13	812	8
8	14	836	8
9	15	864	14
10	16	3186	13
11	17	1774	11
12	18	1834	17
1	8	548	1
2	9	528	8
3	10	528	8
4	11	1056	8
5	12	896	8
6	13	848	8
7	14	902	8
8	15	1062	8
9	16	1854	15
10	17	1854	10
11	18	2014	17
1	9	588	8
2	10	728	8
3	11	858	8
4	12	1128	8
5	13	894	8
6	14	922	8
7	15	1104	8
8	16	2664	8
9	17	1866	13
10	18	1950	17
1	10	728	8
2	11	1128	8
3	12	1056	8
4	13	1056	8
5	14	976	8
6	15	1108	8
7	16	2502	8
8	17	1902	8
9	18	1914	17

i	j	m_{ij}	v_{ij}
1	11	1078	8
2	12	1228	8
3	13	1074	8
4	14	1194	8
5	15	1170	8
6	16	2370	8
7	17	1998	8
8	18	2118	17
1	12	1248	8
2	13	1164	2
3	14	1137	13
4	15	1444	8
5	16	2500	8
6	17	2038	8
7	18	2142	17
1	13	1246	8
2	14	1298	8
3	15	1318	8
4	16	3250	8
5	17	2078	5
6	18	2134	17
1	14	1328	8
2	15	1532	8
3	16	2512	8
4	17	2198	4
5	18	2198	17
1	15	1522	8
2	16	3202	8
3	17	2266	8
4	18	2486	17
1	16	2852	8
2	17	2326	2
3	18	2338	17
1	17	2426	1
2	18	2566	17
1	18	2546	17

Find the optimal order for multiplying $M_1 \times M_2 \times ... \times M_{18}$. Rather than using parentheses to show the optimal order, draw a parse tree. For example, the parse tree corresponding to $((M_1 \times ((M_2 \times M_3) \times (M_4 \times M_5))) \times M_6) \times (M_7 \times M_8)$ would be as shown at right.



L. Consider the weighted digraph at right. Edges not shown have weight ∞. In the terminology of the all-pairs shortest path algorithm for digraphs (See handout on web site.), find $d^{k}_{1,10}$ for $k = 0, 1, 2, ..., 9, 10$, and find $short_k(1,10)$ for all values of k for which $d^{k}_{1,10} < \infty$.

Recall that $short_k(i,j)$ is a path from i to j that has minimal length, subject to the constraint that any intermediate vertices on the path lie in the set $\{1,2,...,k\}$, and that d^{k}_{ij} is the length of $short_k(i,j)$.