

Finding the k^{th} Smallest Element of an Array a of Size n in Expected Linear Time: Example

$a = (26, 51, 40, 14, 77, 59, 32, 82, 63, 45, 37, 89, 78, 57, 22, 60, 99, 74, 34, 19, 53, 48, 39, 95, 87, 11, 92, 73, 68, 86, 65)$,
 $n = 31$, $k = 21$.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
26	51	40	14	77	59	32	82	63	45	37	89	78	57	22	60	99	74	34	19	53	48	39	95	87	11	92	73	68	86	65

$left = 1$, $right = 31$, $partition(a, 1, 31)$ chooses 60 as pivot and returns final pivot position 17.

34	51	40	14	11	59	32	39	48	45	37	53	19	57	22	26	60	74	99	78	89	63	82	95	87	77	92	73	68	86	65
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$left = 18$, $right = 31$, $partition(a, 18, 31)$ chooses 95 as pivot and returns final pivot position 30.

34	51	40	14	11	59	32	39	48	45	37	53	19	57	22	26	60	86	65	78	89	63	82	74	87	77	92	73	68	95	99
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$left = 18$, $right = 29$, $partition(a, 18, 29)$ chooses 82 as pivot and returns final pivot position 25.

34	51	40	14	11	59	32	39	48	45	37	53	19	57	22	26	60	77	65	78	68	63	73	74	82	87	92	86	89	95	99
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$left = 18$, $right = 24$, $partition(a, 18, 24)$ chooses 68 as pivot and returns final pivot position 20.

34	51	40	14	11	59	32	39	48	45	37	53	19	57	22	26	60	63	65	68	77	78	73	74	82	87	92	86	89	95	99
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$left = 21$, $right = 24$, $partition(a, 21, 24)$ chooses 78 as pivot and returns final pivot position 24.

34	51	40	14	11	59	32	39	48	45	37	53	19	57	22	26	60	63	65	68	74	77	73	78	82	87	92	86	89	95	99
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$left = 21$, $right = 23$, $partition(a, 21, 23)$ chooses 77 as pivot and returns final pivot position 23.

34	51	40	14	11	59	32	39	48	45	37	53	19	57	22	26	60	63	65	68	73	74	77	78	82	87	92	86	89	95	99
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$left = 21$, $right = 22$, $partition(a, 21, 22)$ chooses 73 as pivot and returns final pivot position 21.

34	51	40	14	11	59	32	39	48	45	37	53	19	57	22	26	60	63	65	68	73	74	77	78	82	87	92	86	89	95	99
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The 21st smallest element is 73.