CS / MCS 401 Week #5 Exercises (Spring 2008)

Page 86, Exercise 4.4, parts (a), (c), (e), (h).

(For those parts where the Master Theorem, or the extension to the Master theorem given in class, is applicable, quote it to obtain a solution of the form $T(n) = \Theta(g(n))$, where $g(n)$ is as simple as possible (i.e., $g(n)$ should not be $3n^2$ or n^2+2n; use n^2 instead). When you quote the Master theorem, give the values of a, b, $E = \log_b(a)$, and $f(n)$; see page 73 of the text, or the handout on the Master Theorem.). If the Master Theorem is not applicable, solve the recurrence directly, again obtaining a solution of the form $T(n) = \Theta(g(n))$.

Exercise H. Consider the recurrence

$$C(n) = C(0.8n) + C(0.5n) + C(0.2n) + n, \quad C(1) = 1,$$

ignoring the fact that $0.8n$, $0.5n$, and $0.2n$ may not be integers. If the recurrence has a solution of the form

$$C(n) = a\,n^b + c\,n + d$$

for some real numbers a, b, c, and d, what must the value of b be? Compute your answer to two decimal places.

Exercise I. How many inversions are there in the array

a = (41 16 74 33 66 54)?

How many comparisons would straight insertion sort perform in sorting this array? How many exchanges would it perform?

Exercise J. Decide whether each strict partial order $\prec$ on the set S is a strict weak order. Recall that a strict weak order is a strict partial order in which the relation ~ defined by

$$a \sim b \text{ if } \textbf{not}(a \prec b) \textbf{ and not}(b \prec a)$$

is an equivalence relation.[1] If $\prec$ is a strict weak order, give a simple, intuitive description of the equivalence classes of the equivalence relation ~ defined above. (You need not prove that $\prec$ is a strict weak order.) If $\prec$ is a not strict weak order, give a counterexample.

i) S = X–Y plane, $(x,y) \prec (u,v)$ if $x < u$ and $y < v$.

ii) S = X–Y plane, $(x,y) \prec (u,v)$ if $x^2 + 4y^2 < u^2 + 4v^2$.

iii) S = X–Y plane, $(x,y) \prec (u,v)$ if $x - y < u - v$.

iv) S = (X–Y plane), $(x,y) \prec (u,v)$ if $x < u - 1$.

v) S = real numbers, $a \prec b$ if $\lfloor a \rfloor < \lfloor b \rfloor$.

vi) S = real numbers, $a \prec b$ if $a - \lfloor a \rfloor < b - \lfloor b \rfloor$.

vii) S = positive integers, $a \prec b$ if a is a proper divisor of b (i.e., b/a is an integer different from 1).

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[1] To demonstrate that ~ is an equivalence relationship, it is sufficient to show that ~ transitive. Reflexivity and symmetry automatically hold for this particular relation.