CS / MCS 401 Week #6-7 Exercises (Spring 2008)

Exercises 6.3-1 and 6.3-2

Exercise 6.4-1 (As in Fig 6.4, start with the heap just after is has been constructed by *build-max-heap*(). **You may limit yourself to illustrating the first two steps.** One step consists of removing the root from the heap and then restoring the remaining elements to a heap.)

Exercise 6.5.7

Exercise 8.1-3

Exercise K.

a) What is the lower bound (as obtained from a decision tree argument) on the number of comparisons required, in the worst case, to sort an array of size 5 with distinct elements?

b) Could an algorithm that achieves the lower bound in (a) perform only 4 comparisons for two or more inputs, i.e., could the decision tree have two or more leaves at depth 4? Why or why not?

Exercise L. Using Stirling's formula, show that $\binom{n}{n/2} \approx 2^{n-\lg(n)/2-0.326}$. Use this result and a decision-tree argument to obtain a lower bound for the number of comparisons required, in the worst case, to merge two sorted $n/2$ element arrays into a sorted n-element array. Assume that the elements of the merged sequence are distinct.

Exercise M. How many heaps may be formed with 5 distinct elements (say with the integers 1,2,3,4,5)? Draw all such heaps. (If two heaps differ only in that the two elements at maximal depth are exchanged, draw just one of the heaps and indicate that the other is formed by exchange.)

Exercise N. Use your result in Exercise M and decision-tree arguments to obtain a lower bounds for the number of comparisons required, in the worst case, to

a) transform an arbitrary array of size 5 (distinct elements) into a heap.

b) transform a heap of size 5 (distinct elements) into a sorted array.