exam01sample.mw revised 20080212 JL
Maple 10 Worksheet for Problems in Math 165 - Calculus for Business.
% is the Maple notation for the preceding result
First load plots and student:

```
> restart:with( student):with (plots):
```

1. Find the composite function f(2x-3), where

```
f_1:=proc(x);
    1/x + x;
end proc:`f_1(x)`:=f_1(x);
`Ans_1 = f_1(2 x - 3)`:=f_1(2*x - 3);normal(%);
```

$$f\_1(x) := \frac{1}{x} + x$$

$$Ans\_1 = f\_1(2\,x - 3) := \frac{1}{2\,x-3} + 2\,x-3$$

$$\frac{2\left(5+2\,x^2-6\,x\right)}{2\,x-3} \qquad (1)$$

2. At a certain factory, the total cost of manufacturing q units during the daily production run is
C(q) = q^ 2 + 2 g + 297 dollars.
On a typical workday, q(t) = 17t units are manufactured during the first t hours of a production run. How many dollars are spent during the first 3 hours of production?

```
> C:= proc(q);
     q^2 + 2*q + 297;
  end proc:`C(q)`:= C(q);
  q:= proc(t);
     17*t;
  end proc:`q(t)`:= q(t);
  `C(q(3))`:=C(q(3));
```

$$C(q) := q^2 + 2\,q + 297$$

$$q(t) := 17\,t$$

$$C(q(3)) := 3000 \qquad (2)$$

3. True or false: The graphs of and y = x intersect at (1, 1) only.

```
> x_intersection:=solve(x = 1/x,x);
```

$$x\_intersection := 1, -1 \qquad (3)$$

4. Write an equation for the line through (3, 0) with slope 2.
A) y = 2x-6
B) y = 2x-3
C) y = 2x + 6
D) y = 2x + 3

```
> PtSlope:= proc(y,x,x0,y0,m)
      local eqn:
      description `line y = y0 + m*(x - x0)`:
      eqn:= y = y0 + m*(x - x0);
```

```
        eqn;
    end proc:`PtSlope equation`:=PtSlope(y,x,x0,y0,m);
    Ans_4:= PtSlope(y,x,3,0,2);
```

$$PtSlope\ equation := y = y0 + m\ (x - x0)$$

$$Ans\_4 := y = 2\,x - 6 \quad (4)$$

5. Since the beginning of the year, the price of a carton of eggs has been rising at a constant rate of 1.5 cents per month. By May 1, the price had reached 90 cents per carton. Express the price of eggs as a function of time and determine the price at the beginning of the year.
Line with point (4,.90) and slope 1.5
Let t be the time (in months) since the beginning of the year.

```
> Ans_5:=PtSlope(P,t,4,.90,.015);`p(Jan 1)`:=PtSlope(P,0,4,.90,.015)
  ;
```

$$Ans\_5 := P = 0.840 + 0.015\ t$$

$$p(Jan\ 1) := P = 0.840 \quad (5)$$

6. A company makes a certain product for $4 each and sells it for $8. If the company has overhead expenses of $10,000 per year, how many of its products must be made and sold to break even?A) 10,000B) 20,000C) 40,000D) 2,500

```
> Ans_6:=solve(10000 + 4*x = 8*x,x);
```

$$Ans\_6 := 2500 \quad (6)$$

7. A manufacturer's total cost consists of a fixed overhead of 300 plus production costs of 30 per unit. Express the total cost in dollars as a function of the number of units produced.
Line through (0,300) slope 30

```
> Ans_7:=PtSlope(C,q,0,300,30);
```

$$Ans\_7 := C = 300 + 30\ q \quad (7)$$

8. Find the limit: A) 0B) -2C) does not existD) -1

```
> f_8:= proc(x);
        (x+2)/(x^2 - 4);
    end proc:
    `f_8(x)`:=f_8(x);
    `f_8 simplified`:=simplify(f_8(x));
    Ans_8:=limit(f_8(x), x = 2);
```

$$f\_8(x) := \frac{x+2}{x^2-4}$$

$$f\_8\ simplified := \frac{1}{x-2}$$

$$Ans\_8 := undefined \quad (8)$$

9. For which value of x is the following function not continuous: A) 1B) 2C) 0D) -2
Maple uses evalb (evaluate Boolean) to describe true/false

```
> f_9:=proc(x);
```

```
    piecewise(x < 2,x-2,x=2,1,x>2,2-x);
  end proc;
  `f_9(x)`:=f_9(x);
  `limit at x = 2`:=limit(f_9(x),x=2);
  Ans_9:=evalb(limit(f_9(x),x=2)=f_9(2));
```

$$f\_9 := \textbf{proc}(x)\ piecewise(x < 2, x-2, x=2, 1, 2 < x, 2-x)\ \textbf{end proc}$$

$$f\_9(x) := \begin{cases} x-2 & x < 2 \\ 1 & x = 2 \\ 2-x & 2 < x \end{cases}$$

$$limit\ at\ x = 2 := 0$$

$$Ans\_9 := false \qquad (9)$$

10. Find the limit as of f(x) where

```
> f_10:= proc(x);
    piecewise(x <= 4, x^2,x> 4, x+3);
  end proc:
  `f_10(x)`:=f_10(x);
  Ans_10:=limit(f_10(x),x=4,left);
```

$$f\_10(x) := \begin{cases} x^2 & x \le 4 \\ x+3 & 4 < x \end{cases}$$

$$Ans\_10 := 16 \qquad (10)$$

11. The derivative of f(t) = 1/t^2 is

```
> f_11:= proc(t);
   1/t^2;
  end proc:
  `f_11(t)`:=f_11(t);
  `f_11'(t)`:=diff(f_11(t),t);
```

$$f\_11(t) := \frac{1}{t^2}$$

$$f\_11'(t) := -\frac{2}{t^3} \qquad (11)$$

12. The equation of the line tangent to the graph of at x = 2 is

A)y = 7x-4 B)y = 7x-422 C)y = 7x-2 D) y = 7x-144

```
f_12:= proc(x);
    x^2 + 3 * x;
end proc:
`f_12(x)`:=f_12(x);
```

```
d_dx_f_12(x):= diff(f_12(x),x);
slope_12:=eval(d_dx_f_12(x),x=2);
Ans_12:=PtSlope(y,x,2,f_12(2),slope_12);
```

$$f\_12(x) := x^2 + 3x$$

$$d\_dx\_f\_12(x) := 2x + 3$$

$$slope\_12 := 7$$

$$Ans\_12 := y = -4 + 7x \qquad (12)$$

13. True or false: The tangent to the graph of at x = 2 has slope of .

```
> f_13:=proc(x);
     sqrt(x) + 3;
  end proc:
  `f_13(x)`:= f_13(x);
  d_dx_f_13:=proc(x);
   diff(f_13(x),x);
  end proc:
  `f_13``(x)`:=d_dx_f_13(x);
  `slope at x = 2`:=eval(%,x=2);
  Ans_13:= evalb(eval(d_dx_f_13(x),x=3) = 1/2);
```

$$f\_13(x) := \sqrt{x} + 3$$

$$f\_13`(x) := \frac{1}{2\sqrt{x}}$$

$$slope\ at\ x = 2 := \frac{1}{4}\sqrt{2}$$

$$Ans\_13 := false \qquad (13)$$

14. Differentiate

```
> f_14(x):= x^8 + 2;
  Ans_14:= diff(f_14(x),x);
```

$$f\_14(x) := x^8 + 2$$

$$Ans\_14 := 8x^7 \qquad (14)$$

15. Differentiate:

A)

```
> f_15(x):= x^9 + 5;
  Ans_15:= diff(f_15(x),x);
```

$$f\_15(x) := x^9 + 5$$

$$Ans\_15 := 9x^8 \qquad (15)$$

16. Differentiate: .

```
> f_16(x):= root[7](x) - 1/sqrt(x);
  Ans_16:= diff(f_16(x),x);
```

$$f\_16(x) := x^{1/7} - \frac{1}{\sqrt{x}}$$

$$Ans\_16 := \frac{1}{7\,x^{6/7}} + \frac{1}{2\,x^{3/2}} \qquad (16)$$

17. True or false: Differentiating gives .

```
> f_17(x):= x^2 - 5*x +1;
  d_dx_f17(x):= diff(f_17(x),x);
  Ans_17:=evalb(d_dx_f17(x)=2*x^1);
```

$$f\_17(x) := x^2 - 5\,x + 1$$

$$d\_dx\_f17(x) := 2\,x - 5$$

$$Ans\_17 := false \qquad (17)$$

18. What is the rate of change of f(t) =(2t-9)/ (t+4) with respect to t when t = 13?

```
> f_18(t):= (2*t -9)/(t+4);
  d_dt_f18(t):= diff(f_18(t),t);
  `f_18'(t)`:=normal(%);
  Ans_18:=eval(%,t=13);
```

$$f\_18(t) := \frac{2\,t-9}{t+4}$$

$$d\_dt\_f18(t) := \frac{2}{t+4} - \frac{2\,t-9}{(t+4)^2}$$

$$f\_18'(t) := \frac{17}{(t+4)^2}$$

$$Ans\_18 := \frac{1}{17} \qquad (18)$$

19. When toasters are sold for p dollars apiece, local consumers will buy D(p) = 57600/p toasters a month.
It is estimated that t months from now, the price of the toasters will be p(t) = 0.03 t^(3/2) + 22.08 dollars.
Compute the rate at which the monthly demand for the toasters will be changing with respect to time 16 months from now.
Composite functions are tricky in Maple.

```
> D_19(p):= 57600/p;
  p_19(t):= 0.03 * t^(3/2) + 22.08;
  D(t):=eval(D_19(p),p=p_19(t));
  d_dt_D(t):=diff(D(t),t);
  Ans_19:=eval(d_dt_D(t),t=16.);
```

$$D\_19(p) := \frac{57600}{p}$$

$$p\_19(t) := 0.03\,t^{3/2} + 22.08$$

$$D(t) := \frac{57600}{0.03\, t^{3/2} + 22.08}$$

$$d\_dt\_D(t) := -\frac{2592.000000 \sqrt{t}}{\left(0.03\, t^{3/2} + 22.08\right)^2}$$

$$Ans\_19 := -18.00000000 \qquad (19)$$

Another method more closely related to the chain rule:

```
> D:=proc(p);
     57600/p:
  end proc; `D(p)`:=D(p);
  p:=proc(t);
     0.03 * t^(3/2) + 22.08;
  end proc: `p(t)`:=p(t);
  `D(p(t))`:=D(p(t));
  dD_dt:=proc(t);
     diff(D(p(t)),t);
  end proc:`dD_dt`:= dD_dt(t);
  Answer:=eval(%,t=16);`Simplified Answer`:=simplify(%);
```

$$D := \textbf{proc}(p)\ 57600/p\ \textbf{end proc}$$

$$D(p) := \frac{57600}{p}$$

$$p(t) := 0.03\, t^{3/2} + 22.08$$

$$D(p(t)) := \frac{57600}{0.03\, t^{3/2} + 22.08}$$

$$dD\_dt := -\frac{2592.000000 \sqrt{t}}{\left(0.03\, t^{3/2} + 22.08\right)^2}$$

$$Answer := -\frac{2592.000000 \sqrt{16}}{\left(0.48 \sqrt{16} + 22.08\right)^2}$$

$$Simplified\ Answer := -18. \qquad (20)$$

20. An efficiency study of the morning shift at a certain factory indicates that an average worker arriving on the job at 8:00 A.M. will have assembled transistor radios x hours later. Approximately how many radios will the worker assemble between 9:00 and 9:30 A.M.?

A) approximately 12 radios
B) approximately 360 radios
C) approximately 6 radios
D) approximately 5 radios

```
> Q_20:= proc(x);
```

```
    -x^3 + 9*x^2 - 3*x;
end proc:`Q_20(x)`:=Q_20(x);
Rate_20(x):= diff(Q_20(x),x);
`Rate at x = 1`:= eval(Rate_20(x),x=1);
Estimate:=%*(.5);
Actual:=Q_20(1.5) - Q_20(1.0);
```

$$Q\_20(x) := -x^3 + 9x^2 - 3x$$

$$Rate\_20(x) := -3x^2 + 18x - 3$$

$$Rate\ at\ x = 1 := 12$$

$$Estimate := 6.0$$

$$Actual := 7.375 \quad (21)$$

21. Find , where .

Problem 21

Maple has a procedure implicitdiff which finds dy/dx by implicit differentiation.

I (JL) call on some rather obscure Maple functions to perform implicit differentiation.

There is actually a general method using partial derivatives

```
> restart:with(student):#Don't know why I do this
`dy/dx`:=implicitdiff((x - 3*y)^3 = y+5,y,x);
f_21_left:=proc(x,y);
    (x - 3*y)^3;
end proc:
`f_21_left(x,y)`:=f_21_left(x,y);
f_21_right:=proc(x,y);
    y+5;
end proc:
`f_21_right(x,y)`:=f_21_right(x,y);
Ans_21:=implicitdiff(f_21_right(x,y)= f_21_left(x,y),y, x);
```

$$dy/dx := \frac{3\left(x^2 - 6xy + 9y^2\right)}{1 + 9x^2 - 54xy + 81y^2}$$

$$f\_21\_left(x,y) := (x - 3y)^3$$

$$f\_21\_right(x,y) := y + 5$$

$$Ans\_21 := \frac{3\left(x^2 - 6xy + 9y^2\right)}{1 + 9x^2 - 54xy + 81y^2} \quad (22)$$

What is happening:

I (JL) call on some rather obscure Maple functions to perform implicit differentiation.

```
> ddx_eqn_21:=proc(x);
      diff((x - 3*y(x))^3 - y(x) -5,x);
```

```
      convert(%,D);
  end proc;
> first:=ddx_eqn_21(x);
  solve(first,D(y)(x));
```

$$ddx\_eqn\_21 := \textbf{proc}(x)\ diff((x-3*y(x))\hat{}3-y(x)-5, x);\ convert(`\%`, \mathrm{D})\ \textbf{end proc}$$

$$first := 3\ (x-3\ y(x))^2\ (1-3\ \mathrm{D}(y)(x))-\mathrm{D}(y)(x)$$

$$\frac{3\left(x^2-6\,x\,y(x)+9\,y(x)^2\right)}{1+9\,x^2-54\,x\,y(x)+81\,y(x)^2} \tag{23}$$

Problem 21 using partial derivatives
The method relies on the chain rule for partial derivative: dF(x,y) = (DF/Dx)dx + (DF/Dy)dy so that if F(x,y) constant,
dy/dx = - (DF/Dx)/(DF/Dy)

```
F_21:= proc(x,y);
   (x - 3*y)^3 - y -5;
end proc;
d_dx_F_21(x,y):= diff(F_21(x,y),x);
d_dy_F_21(x,y):= diff(F_21(x,y),y);
 -(d_dx_F_21(x,y)/d_dy_F_21(x,y));
```

$$F\_21 := \textbf{proc}(x, y)\ (x-3*y)\hat{}3-y-5\ \textbf{end proc}$$

$$d\_dx\_F\_21(x, y) := 3\ (x-3\ y)^2$$

$$d\_dy\_F\_21(x, y) := -9\ (x-3\ y)^2-1$$

$$-\frac{3\ (x-3\ y)^2}{-9\ (x-3\ y)^2-1} \tag{24}$$

Using procedures

```
> F_21:= proc(x,y);
     (x - 3*y)^3 - y -5;
  end proc:
  `F_21(x,y)`:=F_21(x,y);
  d_dx_F_21:= proc(x,y);
       diff(F_21(x,y),x);
  end proc:`d_dx_F_21(x,y)`:=d_dx_F_21(x,y);
  d_dy_F_21:= proc(x,y);
      diff(F_21(x,y),y);
  end proc:`d_dy_F_21(x,y)`:=d_dy_F_21(x,y);
  dy_dx:= proc(x,y):
       -(d_dx_F_21(x,y)/d_dy_F_21(x,y));
```

```
end proc:`dy_dx(x,y)`:=dy_dx(x,y);
```

$$F\_21(x,y) := (x-3\,y)^3 - y - 5$$

$$d\_dx\_F\_21(x,y) := 3\,(x-3\,y)^2$$

$$d\_dy\_F\_21(x,y) := -9\,(x-3\,y)^2 - 1$$

$$dy\_dx(x,y) := -\frac{3\,(x-3\,y)^2}{-9\,(x-3\,y)^2-1} \tag{25}$$

```
> `dy/dx`:=implicitdiff((x - 3*y)^3 = y+5,y,x);
```

$$dy/dx := \frac{3\left(x^2-6\,x\,y+9\,y^2\right)}{1+9\,x^2-54\,x\,y+81\,y^2} \tag{26}$$

```
>
```