2009testtwosample.mw
Maple 10 Worksheet for Problems in Math 165 - Calculus for Business.
N.B. These problems are a lot easier to do "by hand"
First load plots and student:

```
> restart: with( student):with (plots):setoptions(thickness = 2,
  xtickmarks=5,
  ytickmarks=5):
  with(plottools):with(RealDomain):
```

N.B. A Maple command such as eval(f(x),x=2) is the instruction
``Evaluate f(2)" or
``evaluate the function f(x) at x = 2."
a:= b assigns the value 'b' to the name 'a'
Use a semicolon ; to execute a 'Maple Command' and display the output.
Using a colon : executes a command without displaying the output.
I (JL) usually write functions as a procedure such as the "x square funtion:
square_function:= proc(x);x^2 ; end proc;
Other ways to write functions:
square_function:= x -> x^2;
`%` is the last computed expression (Simlar to ANS on your calculator).

1. Find the intervals of increase and decrease for
14* x^3 + 0*x^2 - 1512*x -4
Find dy/dx and solve > 0, < 0

```
> f_1:= proc(x);14* x^3 + 0*x^2 -  1512*x -4 ; end proc:`f_1(x) `:=
  f_1(x);
  deriv_f_1:= proc(x);
    diff(f_1(s),s);
    eval(%,s=x);
  end proc:`deriv_f_1(x) `:=deriv_f_1(x);
  critical_nos:= solve(deriv_f_1(x)= 0,x);
> int_INC:= solve(deriv_f_1(x) > 0,x);
  int_DEC:= solve(deriv_f_1(x) < 0,x);
  plot_f_1:= plot(f_1(x), x = -10 .. 10):
  display(plot_f_1);
```

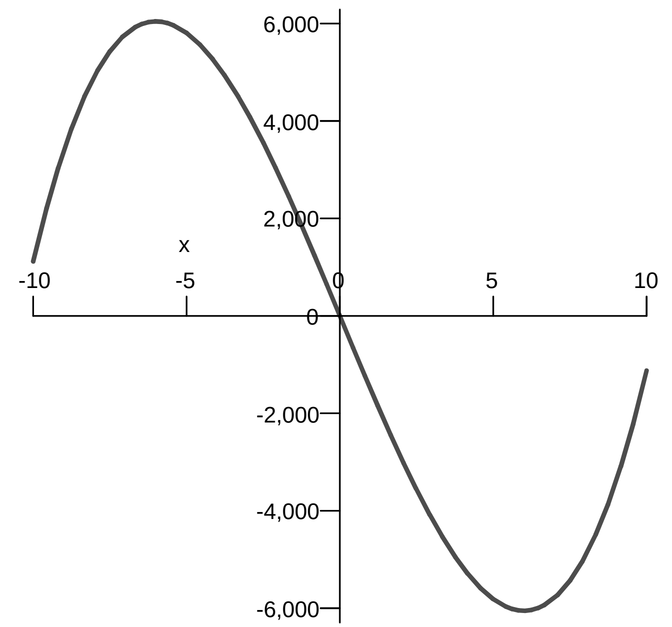
$$f\_1(x) := 14\,x^3 - 4 - 1512\,x$$

$$deriv\_f\_1(x) := 42\,x^2 - 1512$$

$$critical\_nos := 6, -6$$

$$int\_INC := RealRange(-\infty, Open(-6)), RealRange(Open(6), \infty)$$

$$int\_DEC := RealRange(Open(-6), Open(6))$$



2. Maximmize R_2(x) = (24*x - x^2)/(x^2 + 24)

```
> R_2:= proc(x);
  (24*x - x^2)/(x^2 + 24) ;
   end proc:`R_2(x) `:= R_2(x);
  deriv_R_2:= proc(x);
   diff(R_2(s), s );
   eval(%,s=x );
  end proc:`deriv_R_2(x) `:=deriv_R_2(x);`simplified `:=simplify(%);
  crit_no:=solve(deriv_R_2(x)=0,x);answer_2:=`x ` = 4, `R_2(4) ` =
  R_2(4);
  OR:= [[crit_no][2],R_2([crit_no][2])];
  plot_R_2:=plot(R_2(x),x = -0 .. 24, legend = `R_2(x)`):
  display(plot_R_2);
```

$$R\_2(x) := \frac{24\,x - x^2}{x^2 + 24}$$

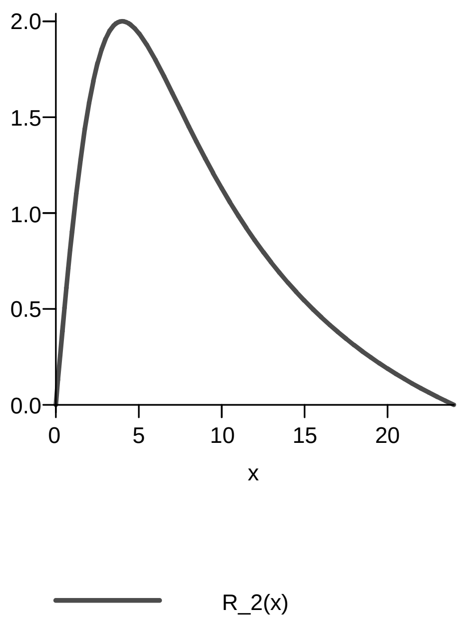
$$deriv\_R\_2(x) := \frac{24-2x}{x^2+24} - \frac{2\,(24x-x^2)\,x}{(x^2+24)^2}$$

$$simplified := -\frac{24\,(x^2-24+2x)}{(x^2+24)^2}$$

$$crit\_no := -6, 4$$

$$answer\_2 := x = 4, R\_2(4) = 2$$

$$OR := [4, 2]$$



3. Critical numbers and classify 1/(x^2 - 8*x + 7)

```
> f_3:= proc(x);1/(x^2 - 8*x + 7)  ; end proc:
  `f_3(x) `:= f_3(x);
  deriv_f_3 := proc(x); diff( f_3  (s), s ); eval(%,s= x); end proc:
  `deriv_f_3 `:= deriv_f_3(x);
  vertical_asymptotes:= solve( x^2 - 8*x + 7 =0 ,x);
  crit_nos:= solve(deriv_f_3(x)=0,x);
   plot3:=plot(f_3(x),x = -10 .. 10, y = -10 .. 10, discont = true,
  thickness = 2):
  answer_3:=`x=4 is a rlative maximum - look at the graph`;
  display(plot3);
```

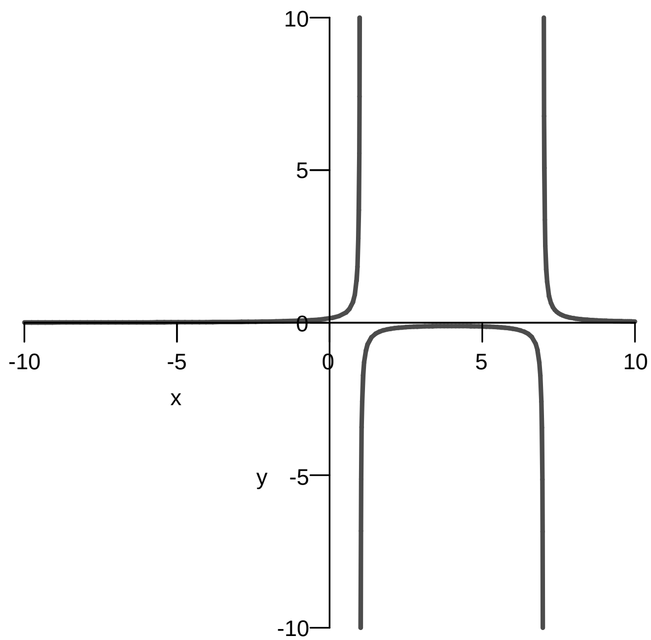
$$f\_3(x) := \frac{1}{x^2-8x+7}$$

$$deriv\_f\_3 := -\frac{2x-8}{(x^2-8x+7)^2}$$

$$vertical\_asymptotes := 7, 1$$

$$crit\_nos := 4$$

answer_3 := x=4 is a rlative maximum - look at the graph



4.

```
> C:= x -> 0.1*x^2 + 20*x + 500;
  A:= x -> C(x)/x;
  deriv_C := proc(x); diff(   C(s), s ); eval(%,s=x ); end proc:
  deriv_A := proc(x); diff(   A(s), s ); eval(%,s=x ); end proc:
  minimum_average_at:= solve({deriv_A(x)=0,x> 0},x);;
```

```
#plot([C(x),A(x)],x = 25 .. 100, discont = true,color=[red,black])
:
```

$$C := x \rightarrow 0.1\ RealDomain\text{:-}\hat{}(x, 2) + 20\,x + 500$$

$$A := x \rightarrow \frac{C(x)}{x}$$

$$minimum\_average\_at := \{x = 70.71067812\} \qquad (1)$$

5. find max min neither x^4 - 4* x^2 + 1

```
> f_5:= proc(x); x^4 - 4* x^2 + 1 ; end proc:`f_5(x) `:=f_5(x);
  deriv_f_5 := proc(x); diff( f_5  (s), s ); eval(%,s=x ); end proc:
  `deriv_f_5(x) `:=deriv_f_5(x);
  deriv2_f_5:= proc(x); diff( f_5  (s), s,s ); eval(%,s=x ); end
  proc:
  `deriv2_f_5(x) `:=deriv2_f_5(x);
  crit_nos:= [solve(deriv_f_5(x) =0, x)];
  `deriv_2_at_crit_nos `:=map(deriv2_f_5, crit_nos);
  answer_5:=`rel MAX at x = 0, rel MIN at \\pm sqrt(2)`;
  plot_f_5:=plot(f_5(x), x = -5 .. 5):
  display(plot_f_5);
```

$$f\_5(x) := x^4 - 4\,x^2 + 1$$

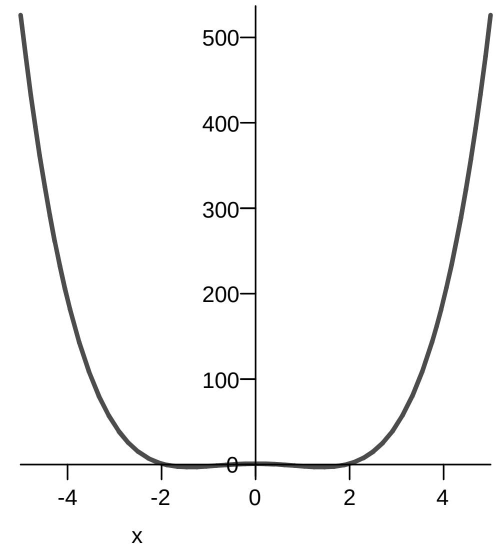
$$deriv\_f\_5(x) := 4\,x^3 - 8\,x$$

$$deriv2\_f\_5(x) := 12\,x^2 - 8$$

$$crit\_nos := \left[0, \sqrt{2}, -\sqrt{2}\right]$$

$$deriv\_2\_at\_crit\_nos := [-8, 16, 16]$$

answer_5 := rel MAX at x = 0, rel MIN at \pm sqrt(2)



6. x^3 +3*x^2 - 9*x +1 concave down

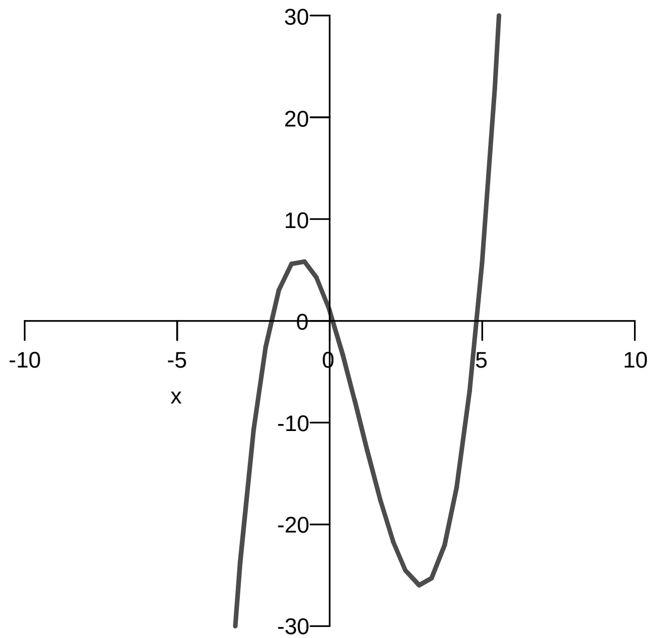
```
> f_6:= proc(x); x^3 -3 *x^2 - 9*x +1 ; end proc:`f_6(x) `:= f_6(x);
  deriv_f_6 := proc(x); diff( f_6  (s), s ); eval(%,s=x ); end proc:
  `deriv_f_6(x) `:=deriv_f_6(x);
  deriv2_f_6 := proc(x); diff( f_6  (s), s,s ); eval(%,s=x ); end
  proc:`deriv2_f_6(x) `:=deriv2_f_6(x);
  `answer_6 - concave down on `:=solve(deriv2_f_6(x) < 0,x);
  plot_f_6:=plot(f_6(x),x = -10 .. 10, -30 .. 30):
  display(plot_f_6);
```

$$f\_6(x) := x^3 - 3\,x^2 - 9\,x + 1$$

$$deriv\_f\_6(x) := 3\,x^2 - 6\,x - 9$$

$$deriv2\_f\_6(x) := 6\,x - 6$$

$$answer\_6\text{ - concave down on } := RealRange(-\infty, Open(1))$$



7 inflection x^4 + 6*x^3 - 24* x^2 + 26; solve y'' =0. actually one should check that sign y'' changes - by factoring in this case

```
f_7:=proc(x); x^4  + 6*x^3 - 24* x^2 + 26 ; end proc:`f_7(x) `:= f_7
(x);
deriv_f_7 := proc(x); diff( f_7  (s), s ); eval(%,s=x ); end
proc:`deriv_f_7(x) `:=deriv_f_7(x);
deriv2_f_7 := proc(x); diff( f_7  (s), s,s ); eval(%,s=x ); end
proc:`deriv2_f_7(x) `:=deriv2_f_7(x);
possible_inflection:=solve(deriv2_f_7(x)=0,x);
concave_up:=solve(deriv2_f_7(x)>0,x);
concave_down:=solve(deriv2_f_7(x)<0,x);
answer_7:=map(x->[x,f_7(x)], [possible_inflection]);
plot_f_7:=plot(f_7(x), x = -10 .. 10):
display(plot_f_7);
```

$$f\_7(x) := x^4 + 6x^3 - 24x^2 + 26$$

$$deriv\_f\_7(x) := 4x^3 + 18x^2 - 48x$$

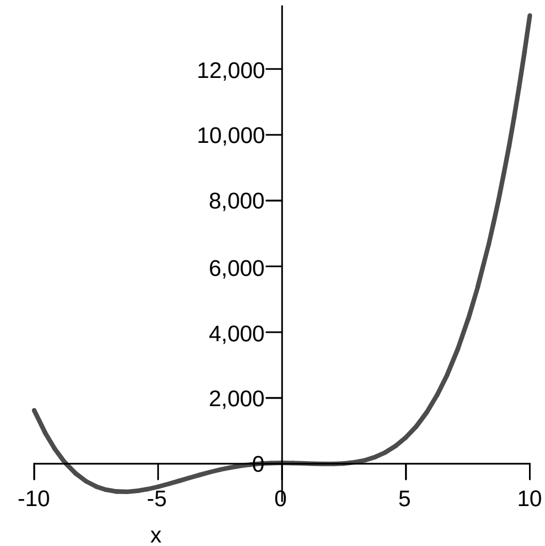
$$deriv2\_f\_7(x) := 12x^2 + 36x - 48$$

$$possible\_inflection := 1, -4$$

$$concave\_up := RealRange(-\infty, Open(-4)), RealRange(Open(1), \infty)$$

$$concave\_down := RealRange(Open(-4), Open(1))$$

$$answer\_7 := [[1, 9], [-4, -486]]$$



8. Critical number x^4 + 6*x^3 - 24* x^2 + 26

```
> f_8:= proc(x);2*x^2 - 8*x +7; end proc:`f_8(x) `:=f_8(x);
  deriv_f_8  := proc(x); diff(   f_8(s), s ); eval(%,s= x); end
  proc:`deriv_f_8(x) `:=deriv_f_8(x);
  deriv2_f_8 := proc(x); diff(   f_8(s), s,s ); eval(%,s= x); end
  proc:`deriv2_f_8(x) `:=deriv2_f_8(x);
  answer_8:= solve(deriv_f_8(x) =0, x);
```

$$f\_8(x) := 2x^2 - 8x + 7$$

$$deriv\_f\_8(x) := 4x - 8$$

$$deriv2\_f\_8(x) := 4$$

$$answer\_8 := 2 \quad (2)$$

9. Analysis of derivative of x^4 + 6*x^3 - 24* x^2 + 26

```
> f_9:= proc(x); -x^3 + 12* x^2 + 144*x + 55; end proc:`f_9(x) `:=
  f_9(x);
  deriv_f_9 := proc(x); diff(   f_9(s), s ); eval(%,s= x); end proc:
     `deriv_f_9(x) `:= deriv_f_9(x);
  deriv2_f_9 := proc(x); diff(   f_9(s), s,s ); eval(%,s= x); end
  proc:
     `deriv2_f_9(x) `:= deriv2_f_9(x);
  answer_max_rate:= maximize(deriv_f_9(t), t = 0 .. 5, location);
  answer_min_rate:= minimize(deriv_f_9(t), t = 0 .. 5, location);
  answer_max_rate_of_rate:=maximize(deriv2_f_9(t), t = 0 .. 5,
  location);
  OR :=maximize(abs(deriv2_f_9(x)), x = 0 .. 5, location);
  total_answer_9:= [t=4,t=0,t=0];plot(f_9(x), x = 0 .. 5):
```

$$f\_9(x) := -x^3 + 12\,x^2 + 144\,x + 55$$

$$deriv\_f\_9(x) := -3\,x^2 + 24\,x + 144$$

$$deriv2\_f\_9(x) := -6\,x + 24$$

$$answer\_max\_rate := 192, \{[\{t=4\}, 192]\}$$

$$answer\_min\_rate := 144, \{[\{t=0\}, 144]\}$$

$$answer\_max\_rate\_of\_rate := 24, \{[\{t=0\}, 24]\}$$

$$OR := 24, \{[\{x=0\}, 24]\}$$

$$total\_answer\_9 := [t=4, t=0, t=0] \quad (3)$$

10. asymptotes 1/(x - 2)

```
> f_10 := proc(x);1/(x - 2); end proc:`f_10(t) `:=f_10(t);
  first_deriv:= diff(f_10(t),t);
  second_deriv:= diff(%, t);
  vertical_asymptote:= solve({x-2 = 0},x);
  horizontal_asymptote:= y = limit(f_10(x),x = infinity);
```

$$f\_10(t) := \frac{1}{t-2}$$

$$first\_deriv := -\frac{1}{(t-2)^2}$$

$$second\_deriv := \frac{2}{(t-2)^3}$$

$$vertical\_asymptote := \{x = 2\}$$

$$horizontal\_asymptote := y = 0 \quad (4)$$

11 abs max t^5 - t^4 on [-1,1]

```
> f_11 := proc(t);t^5 - t^4 ; end proc:`f_11(x) `:=f_11(x);
  first_deriv:= diff(f_11(t),t);`f_11(t) `:= f_11(t);
  critical_numbers:= solve(f_11(t)=0,t);
  endpoints:= {-1,1};
  where_to_look:= {critical_numbers} union endpoints;
  critical_points:=map(x->[x,f_11(x)],where_to_look);
  answer_11:= `abs max value = 0, occurs at t= 0,1`;
  plot(f_11(x), x = -1 .. 1);
```

$$f\_11(x) := x^5 - x^4$$

$$first\_deriv := 5\,t^4 - 4\,t^3$$

$$f\_11(t) := t^5 - t^4$$

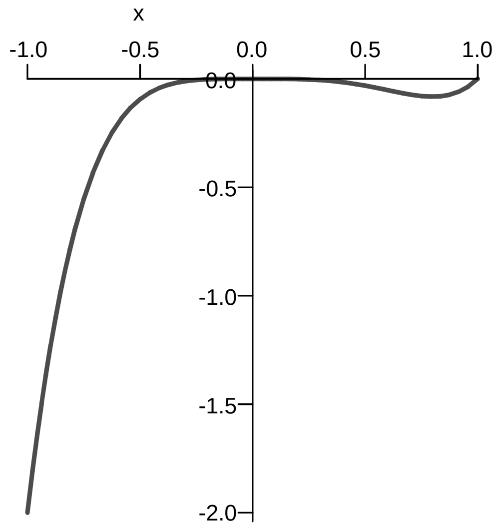
$$critical\_numbers := 1, 0, 0, 0, 0$$

$$endpoints := \{-1, 1\}$$

$$where\_to\_look := \{-1, 0, 1\}$$

$$critical\_points := \{[0, 0], [-1, -2], [1, 0]\}$$

answer_11 := abs max value = 0, occurs at t= 0,1



12.WRONG ANSWER with p = 44-x; Answer OK for p = (45 - x)-- see below

```
> p_12 := proc(x); 44 - x  ; end proc:`p_12(x) `:= p_12(x);
  R_12 := proc(x); x*p_12(x)  ; end proc:`R_12(x) `:= R_12(x);
  C_12 := proc(x); x^2 +5*x +6  ; end proc:`C_12(x) `:= C_12(x);
  P_12 := proc(x); x*p_12(x) - C_12(x) ; end proc:`P_12(x) `:=
  normal(P_12(x));
  MP_12 := proc(x); diff(   P_12(s), s ); eval(%,s= x); end
  proc:`MP_12(x) `:= MP_12(x);
  answer_12:= solve({MP_12(x)=0},x);
> plot(P_12(x), x = 0 .. 20):
```

$$p\_12(x) := 44-x$$

$$R\_12(x) := x\,(44-x)$$

$$C\_12(x) := x^2+5\,x+6$$

$$P\_12(x) := 39\,x-2\,x^2-6$$

$$MP\_12(x) := 39-4\,x$$

$$answer\_12 := \left\{x=\frac{39}{4}\right\} \quad (5)$$

12 again with the function p changed to p = 45-x

```
> p_12m := proc(x); 45 - x  ; end proc:`p_12m(x) `:= p_12m(x);
  R_12m := proc(x); x*p_12m(x)  ; end proc:`R_12m(x) `:= R_12m(x);
  C_12m := proc(x); x^2 +5*x +6  ; end proc:`C_12m(x) `:= C_12m(x);
  P_12m := proc(x); x*p_12m(x) - C_12m(x) ; end proc:`P_12m(x) `:=
  normal(P_12m(x));
  MP_12m := proc(x); diff(   P_12m(s), s ); eval(%,s= x); end
  proc:`MP_12m(x) `:= MP_12m(x);
  answer_12m:= solve({MP_12m(x)=0},x);
> plot(P_12m(x), x = 0 .. 20):
```

$$p\_12m(x) := 45-x$$

$$R\_12m(x) := x\,(45-x)$$

$$C\_12m(x) := x^2+5\,x+6$$

$$P\_12m(x) := 40\,x-2\,x^2-6$$

$$MP\_12m(x) := 40-4\,x$$

$$answer\_12m := \{x=10\} \quad (6)$$

13. max profit

```
> q_13 := proc(p); 260 + ((-4)/12)*(p - 320) ; end proc:`q-13(p) `:=
  q_13(p);
  P_13 := proc(p); (p-40)*q_13(p) ; end proc:`P_13(p) `:= P_13(p);
  deriv_P_13 :=  proc(x); diff(   P_13(s), s ); eval(%,s= x); end
  proc:`deriv_P_13(p) `:= deriv_P_13(p);
  max_profit_1t:=solve( {deriv_P_13(p) =0}, p);
```

$$q\text{-}13(p) := \frac{1100}{3}-\frac{1}{3}\,p$$

$$P\_13(p) := (p-40)\left(\frac{1100}{3}-\frac{1}{3}\,p\right)$$

$$deriv\_P\_13(p) := 380-\frac{2}{3}\,p$$

$$max\_profit\_1t := \{p=570\} \quad (7)$$

14. max revenue train fare

```
> q_14:= proc(p); 600. + ((50)/(-.05))*(p - 1) ; end proc:`q_14(p)
  `:= q_14(p);
  R_14 := proc(p); p*q_14(p) ; end proc:`R_14(p) `:= R_14(p);
  deriv_R_14 :=  proc(x); diff(   R_14(s), s ); eval(%,s= x); end
  proc:`deriv_R_14(p) `:= deriv_R_14(p);
  max_profit_at:=solve( {deriv_R_14(p) =0}, p);
```

$$q\_14(p) := 1600.000000-1000.000000\,p$$

$$R\_14(p) := p\,(1600.000000 - 1000.000000\,p)$$
$$deriv\_R\_14(p) := 1600.000000 - 2000.000000\,p$$
$$max\_profit\_at := \{p = 0.8000000000\} \quad (8)$$

15. x>0, y > 0, x+y = 10, maximize xy^2

```
> constraint:= x + y =10;
  M_15_2 := proc(x,y); x* (y^2)  ; end proc:`M_15_2(x,y) `:= M_15_2
  (x,y);
  `solve_for y `:=solve(constraint, y);
  M_15 := proc(x); simplify(M_15_2(x,solve(constraint, y))) ; end
  proc:
  `M_15(x) `:= M_15(x);
  deriv_M_15 :=  proc(x); diff(   M_15(x), x ); end proc:`deriv_M_15
  (x) `:= deriv_M_15(x);
  x_answer:=solve({deriv_M_15(x) =0, x > 0, 10 - x >0 },x);
  y_answer:= 10 - 10/3;
```

$$constraint := x + y = 10$$
$$M\_15\_2(x,y) := x\,y^2$$
$$solve\_for\ y := -x + 10$$
$$M\_15(x) := x\,(x-10)^2$$
$$deriv\_M\_15(x) := (x-10)^2 + 2\,x\,(x-10)$$
$$x\_answer := \left\{x = \frac{10}{3}\right\}$$
$$y\_answer := \frac{20}{3} \quad (9)$$

16 maximize average cost; C = (1/8) * x^2 +4*x +200

```
>  C_16:= proc(x); (1/8)  * x^2 +4*x +200;normal(%); end proc:`C_16
  (x) `:=C_16(x);
  A_16:= proc(x); C_16(x)/x; end proc:`A_16(x) `:=A_16(x);
  deriv_A_16:= proc(x);diff(A_16(x),x)  ; end proc:`deriv_A_16(x)
  `:=deriv_A_16(x);
  `OR `:=simplify(%);
  answer_16:= solve({deriv_A_16(x) = 0, x > 0},x);
```

$$C\_16(x) := \frac{1}{8}\,x^2 + 4\,x + 200$$
$$A\_16(x) := \frac{\frac{1}{8}\,x^2 + 4\,x + 200}{x}$$
$$deriv\_A\_16(x) := \frac{\frac{1}{4}\,x + 4}{x} - \frac{\frac{1}{8}\,x^2 + 4\,x + 200}{x^2}$$
$$OR := \frac{1}{8}\,\frac{x^2 - 1600}{x^2}$$
$$answer\_16 := \{x = 40\} \quad (10)$$

17.Wrong Answer for q = (300 - p^2)/60. Answer OK for q = (30000 - p^2)/60.-- see below

```
> q_17 := proc(p);(300 - p^2)/60 ; end proc:`q_17(p) := `:= q_17(p);
  deriv_q_17 :=  proc(x);
     diff( q_17(  s), s );
     eval(%,s= x);
  end proc:
  `deriv_q_17(p) `:= deriv_q_17(p);
  E_17 := proc(p); simplify((p/q_17(p))* deriv_q_17(p))  ; end
  proc:`E_17(p) `:= E_17(p);
  demand_of_unit_elasticity:=solve({E_17(p) = -1,p>0, p<10*sqrt(3)},
  p);
  #elastic_demand:= solve({E_17(p) < -1,p>0, p<10*sqrt(3)},p);
  `answer_17 elastic_demand`:= ` p > 10 `;
```

$$q\_17(p) := \ := 5 - \frac{1}{60}\,p^2$$
$$deriv\_q\_17(p) := -\frac{1}{30}\,p$$
$$E\_17(p) := \frac{2\,p^2}{-300 + p^2}$$
$$demand\_of\_unit\_elasticity := \{p = 10\}$$
$$answer\_17\ elastic\_demand := p > 10 \quad (11)$$

17.again with q =. (30000 - p^2)/60 works -- see below

```
> q_17 := proc(p);(30000 - p^2)/60 ; end proc:`q_17(p) := `:= q_17
  (p);
  deriv_q_17 :=  proc(x);
     diff( q_17(  s), s );
     eval(%,s= x);
  end proc:
  `deriv_q_17(p) `:= deriv_q_17(p);
  E_17 := proc(p); simplify((p/q_17(p))* deriv_q_17(p))  ; end
  proc:`E_17(p) `:= E_17(p);
  demand_of_unit_elasticity:=solve({E_17(p) = -1,p>0, p<100*sqrt(3)}
  ,p);
  #elastic_demand:= solve({E_17(p) < -1,p>0, p<100*sqrt(3)},p);
  `answer_17 elastic_demand`:= ` p > 100 `;
```

$$q\_17(p) := \ := 500 - \frac{1}{60}\,p^2$$

$$deriv\_q\_17(p) := -\frac{1}{30}\ p$$

$$E\_17(p) := \frac{2\ p^2}{-30000 + p^2}$$

$$demand\_of\_unit\_elasticity := \{p = 100\}$$

$$answer\_17\ elastic\_demand := p > 100 \qquad (12)$$

18 Variation of Price - Demand Yield = Price

```
> Y_18 := proc(x);200 + (-2)* (x-40)  ; end proc:`Y_18(x) `:= Y_18(x);
  R_18 := proc(x); x * Y_18(x) ; end proc:`Y_18 (x) `:= Y_18(x);
  answer_18:= maximize(R_18(x),x, location);
```

$$Y\_18(x) := 280 - 2\ x$$

$$Y\_18\ (x) := 280 - 2\ x$$

$$answer\_18 := 9800,\ \{[\{x = 70\},\ 9800]\} \qquad (13)$$

19. 0 < x < 10, x+y =10, maximize x^2*y^2

```
> constraint:= x + y =10;
  M_19_2 := proc(x,y); x^2 * (y^2)  ; end proc:`M_19_2(x,y) `:=
  M_19_2(x,y);
  `solve_for y `:=solve(constraint, y);
  M_19 := proc(x); simplify(M_19_2(x,solve(constraint, y))) ; end
  proc:
  `M_19(x) `:= M_19(x);
  #diff(M_19(x),x);
  #don't know why cannot evaluate
  deriv_M_19 :=  proc(x); diff(   M_19(x), x ); end proc:`deriv_M_19
  (x) `:= deriv_M_19(x);
  x_answer:=solve({deriv_M_19(x) =0, x > 0, 10 - x >0 },x);
  y_answer:= 10 - 5;
```

$$constraint := x + y = 10$$

$$M\_19\_2(x,y) := x^2\ y^2$$

$$solve\_for\ y := -x + 10$$

$$M\_19(x) := x^2\ (x - 10)^2$$

$$deriv\_M\_19(x) := 2\ x\ (x - 10)^2 + 2\ x^2\ (x - 10)$$

$$x\_answer := \{x = 5\}$$

$$y\_answer := 5 \qquad (14)$$

20.future value

```
> I_20:=3000;
  r_20:=.10;
  T_20:=9;
  answer_20:= I_20 * exp(r_20 * T_20);
```

$$I\_20 := 3000$$

$$r\_20 := 0.10$$

$$T\_20 := 9$$

$$answer\_20 := 7378.809333 \qquad (15)$$

21 Wrong Answer for f(1) = 100, multiply by 100 every year. f(3) = 10^6. Answer OK forh f(1) = 10.

```
> f_21(1):= 100;
  `Answer_21 f_21(3) = f_21(1))^3 `:= (f_21(1))^3;
  f_21(1):= 10;
  `Answer_21 f_21(3) = f_21(1))^3 `:= (f_21(1))^3;
```

$$f\_21(1) := 100$$

$$Answer\_21\ f\_21(3) = f\_21(1))\text{^}3 := 1000000$$

$$f\_21(1) := 10$$

$$Answer\_21\ f\_21(3) = f\_21(1))\text{^}3 := 1000 \qquad (16)$$

22. future value

```
> PV_22:=4000;
  r_22:=.08;
  T_22:=11;
  answer_22:= I_22 * exp(r_22 * T_22);
```

$$PV\_22 := 4000$$

$$r\_22 := 0.08$$

$$T\_22 := 11$$

$$answer\_22 := 2.410899706\ I\_22 \qquad (17)$$

23. count :-- half life is six weeks

```
> F_23 := proc(x);(1/2)^(x/6)  ; end proc:`F_23(x) `:= F_23(x);
  answer_23:= F_23(18);
```

$$F\_23(x) := \left(\frac{1}{2}\right)^{\frac{1}{6}x}$$

$$answer\_23 := \frac{1}{8} \qquad (18)$$

24. count: every 100 years multiply by 600/800

```
> F_24 := proc(x);800*(600/800)^(x/100)  ; end proc:`F_24(t) `:=
  F_24(t);
  answer_24:= evalf(F_24(400),7);
```

$$F\_24(t) := 800\ \left(\frac{3}{4}\right)^{\frac{1}{100}t}$$

$$answer\_24 := 253.1250 \qquad (19)$$

25. solve 2*ln(x) - (1/3)*ln(x^2):=4

```
> LHS:=2*ln(x) - (1/3)*ln(x^2):RHS:=4:
  EQN:= LHS = RHS;
  `use ln rules on both sides`:=map(x->simplify(x,ln,symbolic),[LHS,
  RHS]);
  `take exp of both sides` :=map(x->exp(x),%);
  `take 3/4 power of both sides `:= [x,e^3];answer_25:=solve(%[1]=%
  [2],x);
```

$$EQN := 2\ln(x) - \frac{1}{3}\ln(x^2) = 4$$

$$use\ ln\ rules\ on\ both\ sides := \left[\frac{4}{3}\ln(x), 4\right]$$

$$take\ exp\ of\ both\ sides := [x^{4/3}, e^4]$$

$$take\ 3/4\ power\ of\ both\ sides\ := [x, e^3]$$

$$answer\_25 := e^3 \tag{20}$$

26. carbon dating

```
> Age_26 := proc(t);A_0 * (1/2)^(t/5730)  ; end proc:`Age_(t) `:=
  Age_26(t);
  Answer_26:=evalf(solve(Age_26(t) = (1/7)*A_0,t));
```

$$Age\_(t)\ := A\_0\left(\frac{1}{2}\right)^{\frac{1}{5730}t}$$

$$Answer\_26 := 16086.14370 \tag{21}$$

27. 5^*x* = *e*^8`; `*note* ln(*e*) = 1; *take* ln *of both sides* (*easier* **by** *hand*!

```
> LHS:=5^x:RHS:=exp(8):
  EQN:= LHS = RHS;
  `apply ln to both sides `:= map(x -> simplify(ln(x),symbolic),
  [LHS,RHS]);
  answer_27:= solve(%[1]=%[2],x);
```

$$EQN := 5^x = e^8$$

$$apply\ ln\ to\ both\ sides\ := [x\ln(5), 8]$$

$$answer\_27 := \frac{8}{\ln(5)} \tag{22}$$

28. Find the interest rate solve({PV_28* exp(r*T_28) = FV_28},r)

```
> FV_28:=6000;
  PV_28:=3000;
  T_28:=14.;
  answer_28:=solve({PV_28* exp(r*T_28) = FV_28},r);
  `ln(2)/(14) `:= ln(2.) / 14;
```

$$FV\_28 := 6000$$

$$PV\_28 := 3000$$

$$T\_28 := 14.$$

$$answer\_28 := \{r = 0.04951051290\}$$

$$ln(2)/(14)\ := 0.04951051290 \tag{23}$$