

MthT 430 2002 Midterm Common Mistakes

I. Definitions

1. (10 points) Define ...
2. (10 points) Is ...

II. Examples

3. (10 points) Give an example of two functions f and g such that $f \circ g = g \circ f$. Be sure to verify that the domains are the same.

Common Mistake: Not verifying the domains

4. (10 points) Give an example of a function $f(x)$ defined for all real numbers such that $\lim_{x \rightarrow 0} f(x)$ exists but does not equal $f(0)$.

Common Mistake: Not verifying the domain or not redefining the function at $x = 0$ so that it is discontinuous at 0.

5. (20 points) Let

$$F(x) = \sqrt{x^2 - 1},$$
$$G(x) = \sqrt{1 - x^2}.$$

Describe:

- domain (F) and domain (G).

Common Mistake: Not identifying correctly domain (F) = $\{|x| \geq 1\}$.

- domain ($G \circ F$) must be a subset of domain F . Answer: $\{1 \leq |x| \leq \sqrt{2}\}$

III. Proofs

6. (20 points) Show, using only P1 – P9:

For all a , $a \cdot 0 = 0$.

You may abbreviate (*distributive*, ...).

Common Mistakes:

- Starting by assuming the result and manipulating:

$$a \cdot 0 = 0 \tag{1}$$

$$a^{-1} \cdot a \cdot 0 = 0 \cdot a^{-1} \tag{2}$$

$$1 \cdot 0 = 0 \cdot a^{-1} \tag{3}$$

$$0 = 0 \tag{4} \quad \text{RHS assumes result!}$$

If all the steps were justified, one would have to “reverse” to obtain the result. Going from (4) to (3) assumes the result. All that has been shown is: $a \cdot 0 = 0$ implies $0 \cdot a^{-1} = 0$.

- Not using $P9$ (distributive) somewhere.

7. (20 points) Show by mathematical induction or otherwise: (Bernoulli’s Inequality) For all natural numbers $n = 1, 2, \dots$, for $x > -1$,

$$(1 + x)^n \geq 1 + nx.$$

Common Mistakes:

- (Clarity) $P(n)$ is not stated clearly.
- (Minor) Not being explicit as to how the condition $x > -1$ is used in passing from $P(n)$ to $P(n + 1)$
- Writing $P(n) = (1 + x)^n \geq 1 + nx$
- Writing $P(n + 1)$ and manipulating to try to obtain $P(n)$.
- (We won’t do this again!) $(1 + x)^{n+1} = 1^{n+1} + x^{n+1}$.

IV. Qualitative Properties of Functions

8. (30 points) The graph below shows how the height of a liquid in beaker X varies as water is steadily dripped into it. Copy the graph, and *on the same diagram* show the height–volume relationship for the Ink Bottle.

Describe the features of the graph you have drawn. Your description should include . . .

Common Mistakes

- Identifying the domain
- Identifying the maximum height (same as for Beaker X)
- Interchanging the the roles of Height and Volume when saying “faster”, etc.
- Incomplete word descriptions