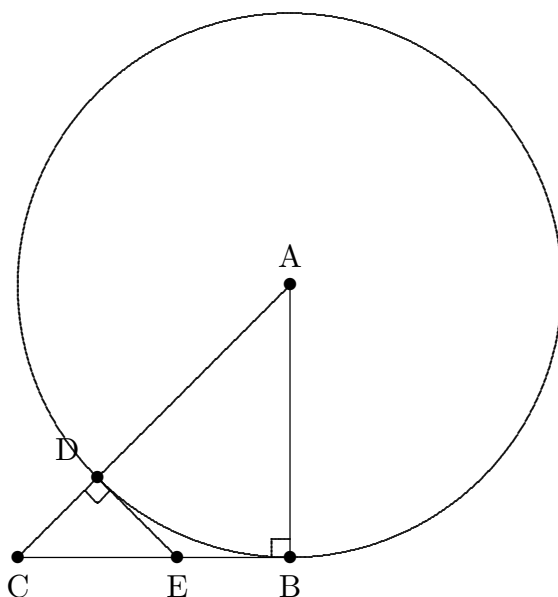


### MthT 430 Apostol's Irrationality

The following proof of the irrationality of  $\sqrt{2}$  is credited to Thomas Apostol. It relies on geometry and the principle of mathematical induction.

Suppose that there are natural numbers  $m$  and  $n$  such that  $m^2 = 2n^2$  and that  $n$  is the smallest natural number for which  $m^2 = 2n^2$ . Draw a circle centered at  $A$  so that  $AB = BC = n$  and  $AC = m$ .



Let  $DE$  be tangent to the circle at  $D$  and  $BC$  be tangent to the circle at  $B$ . Then  $DE = EB = DC = m - n$ . Then  $\triangle DCE$  is a right triangle with sides  $m - n < n$  and hypotenuse  $n - (m - n) = 2n - m$  and  $n$  is not the smallest natural number satisfying  $m^2 = 2n^2$ .

Tom M. Apostol, Irrationality of The Square Root of Two -- A Geometric Proof, American Mathematical Monthly **107**, No. 9 (Nov., 2000), pp. 841-842.