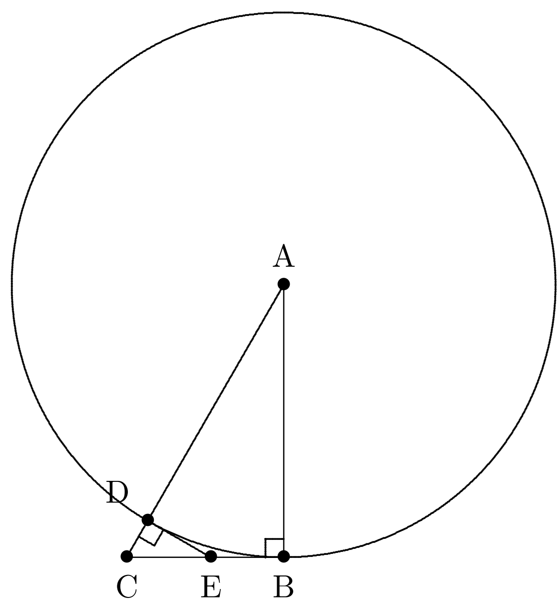
MthT 430 Apostol's Irrationality – Still More

Messrs. Huynh, Meeks, and Wesby, MthT 430 Fall 2005, proposed a simplification of the proof for the irrationality of $\sqrt{N^2-1}$ except in the obvious cases.

Irrationality of $\sqrt{N^2-1}$

Use the same picture:



In the above picture note that $\Delta EDC \simeq \Delta ABC$, so that

$$\frac{CD}{CB} = \frac{DE}{AB} = \frac{CE}{AC}.$$

Also note that $DE = EB$ since $\Delta ADE \simeq \Delta ABE$ – or use the Pythagorean Theorem.

If $q^2\left(N^2-1\right) = p^2$ for natural numbers $q > 1$, p, we may construct ΔABC with integer sides so that

$$\begin{aligned} AB &= q\sqrt{N^2-1}, \\ AC &= qN, \\ CB &= q. \end{aligned}$$

Then

$$
\begin{aligned}
CE &= CD \cdot \frac{AC}{CB} \\
&= \text{integer} \cdot \frac{qN}{q} \\
&= \text{integer.} \\
DE &= EB \\
&= CB - CE \\
&= \text{integer.}
\end{aligned}
$$

Thus we have a smaller triangle with integer sides which is similar to the triangle ABC.

Tom M. Apostol, Irrationality of The Square Root of Two -- A Geometric Proof, American Mathematical Monthly **107**, No. 9 (Nov., 2000), pp. 841-842.