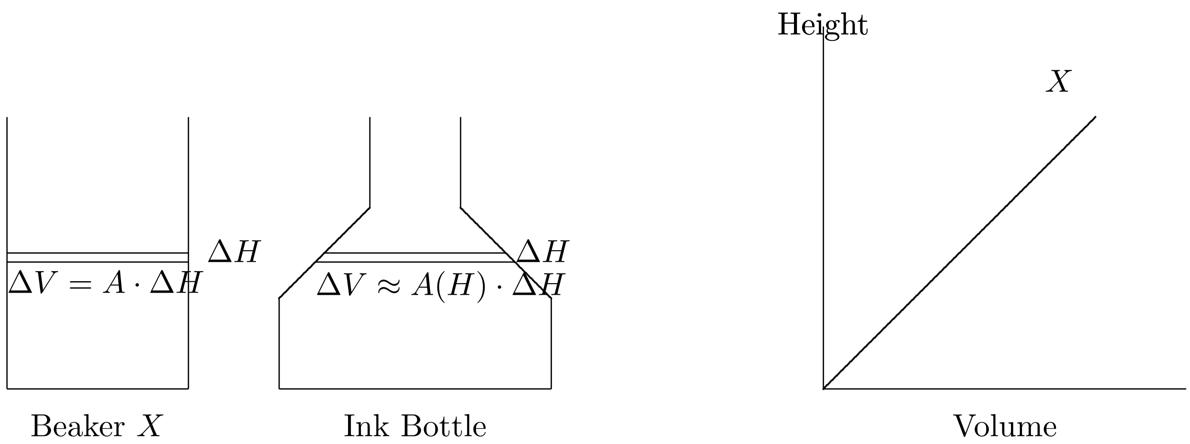
MthT 430 Notes Chapter 5b Geometric Limits

Geometric Limits



Let A or $A(H)$ denote the *cross sectional area* at height H.

For Beaker X,

$$\frac{\Delta H}{\Delta V} = \frac{1}{A},$$
$$\lim_{\Delta V \to 0} \frac{\Delta H}{\Delta V} = \frac{1}{A}.$$

For the Ink Bottle,

$$\frac{\Delta H}{\Delta V} \approx \frac{1}{A(H)},$$
$$\lim_{\Delta V \to 0} \frac{\Delta H}{\Delta V} = \frac{1}{A(H)}.$$

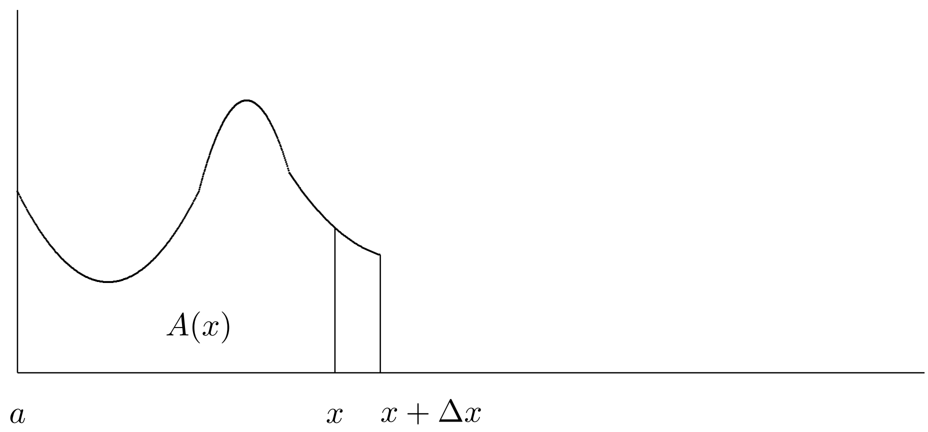
Notice that for a circular flask $A = \pi\,(\text{radius})^2$.

Fundamental Theorem of Calculus

Let $y = f(x)$ be a nonnegative continuous function and let $A(x)$ denote area of the region

$$(t, y) : \begin{cases} a \le t \le x, \\ a \le y \le f(x). \end{cases}$$

This is the region with *first coordinate between a and x, second coordinate between the x–axis and the graph of f.*



Now the change in $A(x)$ as x changes from x to $x + \Delta x$,

$$\Delta A = A(x + \Delta x) - A(x),$$

is the area of the slender region and is

$$\Delta A \approx f(x) \cdot \Delta x;$$

certainly ΔA is between $f(x) \cdot \Delta x$ and $f(x + \Delta x) \cdot \Delta x$. Thus if f is continuous at x,

$$\lim_{\Delta x \to 0} \frac{\Delta A}{\Delta x} = f(x).$$

Identifying area with the integral, this is one form of the Fundamental Theorem of Calculus (FTC):

Theorem. *If f is continuous on $[a, b]$, then for $a < x < b$,*

$$\frac{d}{dx} \int_a^x f(t)\, dt = f(x).$$