

MthT 491 Arithmetic

Motivated by **School Mathematics Project, Book 1**, Cambridge, 1965, [SMP1].

Bases for Integers

Hindu–Arabic System.

How many different numerals do we use in writing down the numbers from 1 to 99? (99 consists of the numeral, 9, written twice.) ... the numbers from 1 to 999?

We use base 10, *denary* numbers or *decimal* arithmetic, and *place value*, so that

$$145 \text{ means } 1 \times 100 + 4 \times 10 + 5 \times 1.$$

100

40

5

145

Other bases:

- Base 12, *duodecimal* arithmetic so that

$$145_{12} \text{ means } 1 \times 144 + 4 \times 12 + 5 \times 1 = 144_{10} + 48_{10} + 5_{10} = 187_{10}.$$

Mixed arithmetic: feet and inches (12 in = 1 ft)

$$1'8'' + 9'6'' = 11'2''.$$

Weeks and days (base 7 for days)

★ (weird)	odd	even
odd		
even		

100

40

1

145

Negative Numbers Chapter 12

For each nonnegative number, a , we associate a positive shift number, $+a$, and a negative shift number, $-a$. We think of these numbers as marching orders, e.g., $+3$ means *3 paces forward*, and $+4$ means *4 paces forward*. Then we can talk about $+3 + -4 = -1$.

Next we map numbers of the form $+a, -b$ to a number line – the position arrived at starting from 0. Call these new guys *shift numbers* also.

”The difference between a shift number a and a shift number b can be described by the shift needed to move from one to the other.” (p. 200). I think this means ” $b - a$ ” is defined as: If b has been ordered what must be ordered to obtain the result of a , i.e., solve $a + ? = b$.

The examples say: the position of $+5$ in relation to the position of -3 is $+8$, since a positive shift of 8 is needed to reach $+5$ from -3 ,

$$\begin{aligned} -3 + +8 &= +5 \\ +5 - -3 &\equiv +8. \end{aligned}$$

Similarly, since

$$\begin{aligned} +5 + -8 &= -3 \\ -3 - +5 &\equiv -8. \end{aligned}$$

Algebraically, since $+a + -a = +0 = -0$ (returning to the same position!), we could apply associativity and commutativity to show that the “rule” is:

At the end, *subtracting* a shift number (from another) is equivalent to reversing the direction of the shift it represents and adding.

$$\begin{aligned} +7 - -3 &= +7 + +3 \\ &= +10. \\ +7 - +3 &= +7 + -3 \\ &= +4. \\ +5 - +8 &= +5 + -8 \\ &= -3. \\ -5 - -2 &= -5 + +2 \\ &= -3. \end{aligned}$$