MthT 491 Contradiction

Contradiction

If A denotes some assertion or collection of assertions, we have a *contradiction* if A is true and A is false – *id est* A is true and $\neg A$, the *negation of* A, are true.

Examples

A is the [mathematical] statement

- All girls are good at mathematics.

The *negation* of A is the [mathematical] statement

- There is some girl who is not good at mathematics.

A theorem

$$A \Rightarrow B$$

is *proved by contradiction* if we show that

$$\neg B \Rightarrow \neg A.$$

Please note that usually the assertion A may contain within itself many definitions and properties not stated explicitly. For example, if A contains the statement

> n is a natural number ...,

and we proved that

> $\neg B$ implies $n < 0$.

we would have a *proof by contradiction* of $A \Rightarrow B$.