The Algebra Symposium: Bouncing Balls

From **Calculus**, Hughes–Hallett, *et al.*:

1. A ball is dropped from a height of 10 feet and bounces. Each bounce is $\frac{3}{4}$ of the height of the bounce before. Thus after the ball hits the floor for the first time, the ball rises to a height of $10\left(\frac{3}{4}\right) = 7.5$ feet, and after the it hits the floor for the second time, the ball rises to a height of $7.5\left(\frac{3}{4}\right) = 10\left(\frac{3}{4}\right)^2 = 5.625$ feet.

(a) Find an expression for the height to which the ball rises after it hits the floor for the n^{th} time.

(b) Find an expression for the total vertical distance the ball has traveled when it hits the floor for the first, second, third, and fourth times.

(c) Find an expression for the total vertical distance the ball has traveled when it hits the floor for the n^{th} time. Express your answer in a closed form.

Hint

$$1 + r + r^2 + \ldots + r^n = \frac{1 - r^{n+1}}{1 - r}.$$

2. You might think that the ball [in the previous problem] keeps bouncing forever since it takes infinitely many bounces.

   Is this true?

Some references from a Google Search for `bounce ratio`:

Geometric Sequences and Series from **Algebra: Themes, Tools, Concepts**, by Anita Wah and Henri Picciotto:

`http://www.picciotto.org/math-ed/seq/geometric.pdf`

Zeno and the Bouncing Ball:

`http://www.sas.org/E-Bulletin/2002-09-20/handsOnPhys/body.html`

The Bounce of the Ball:

`http://wings.avkids.com/Curriculums/Tennis/bounce_summary.html`

Note that the *COR* (Coefficient of Restitution) is not the same as the bounce ratio.