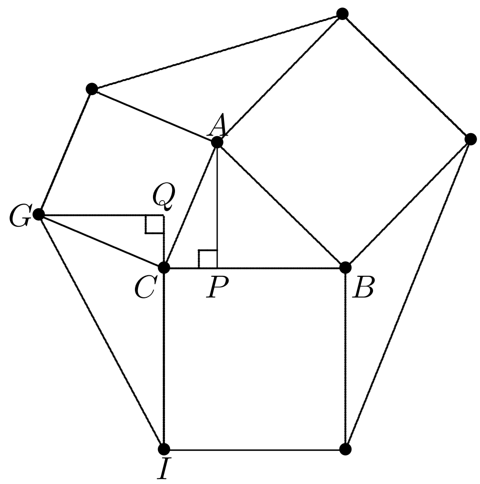
The Algebra Symposium: Geometric Delicacies Discussion

From: Bettina Pedemonte and Elisabetta Robotti, Aspetti linguistici della dimostrazione (Linguistic Aspects of Proof), Notiziario Unione Matematica Italiana, **XXXI**, No. 10, October, 2004, pp. 12–30.

2. ABC is a arbitrary triangle. On the exterior of each side of the triangle, a square is constructed. Joining the free corners of each square, three new triangles are created.

   Compare the area of each of the three new triangles with the area of the triangle ABC.

Discussion: Put some additional labels on the figure.



Compare the areas of ΔABC and ΔGCI.

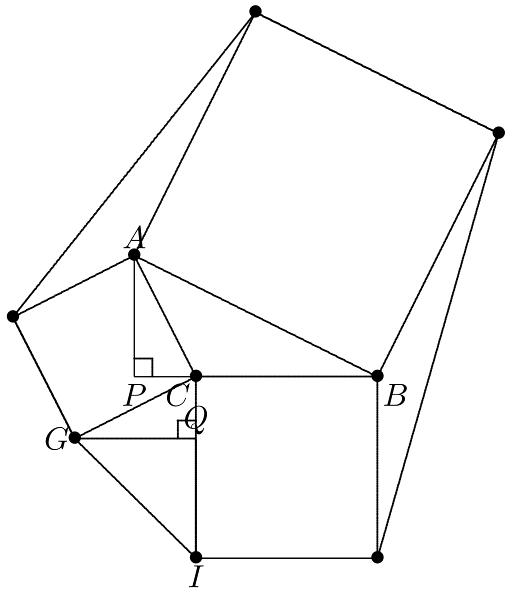
$$\begin{aligned} \text{Area}\,(\Delta ABC) &= \frac{1}{2}bh \\ &= \frac{1}{2}CB \cdot AP, \\ \text{Area}\,(\Delta GCI) &= \frac{1}{2}IC \cdot GQ. \end{aligned}$$

Note that $CB = IC$, and $\Delta GQC \cong \Delta APC$ so that $AP = GQ$. Hence

$$\text{Area}\,(\Delta ABC) = \text{Area}\,(\Delta GCI)\,.$$

More...

One might ask if the argument works if $\angle ACB$ is obtuse (here $\approx 117°$).



Yes!