

PDE WITH LOSS OF DERIVATIVES - Abstract

Joseph J. Kohn

Let X_1, \dots, X_p be complex valued vectorfields in a neighborhood of the origin in \mathbb{R}^n that satisfy the bracket condition at the origin. That is: the Lie algebra generated by the X_1, \dots, X_p evaluated at the origin spans the tangent space at the origin. According to a theorem of Hörmander if the vectorfields are real and satisfy the bracket condition at the origin then the operator

$$E = \sum X_i^* X_i,$$

where the X_i^* are L_2 adjoints of the X_i , is locally hypoelliptic in a neighborhood U of the origin. Furthermore the operator E **gains** derivatives in the sense that u is always smoother than Eu .

When the vectorfields are complex the situation is quite different. In general the operator E **loses** derivatives, in the sense that there exists u for so that Eu is less smooth than u . Amazingly, despite the loss of derivatives, there are examples for which E is hypoelliptic.