NAME:

Open notes, open computer, but closed mouth!

page	1	2	3	4	5	6	7	total
points								
maximum	25	25	25	30	35	35	25	200

1. Consider the polynomial $x^3 + x + 1$ over $\mathbb{Z}_7 = \{0, 1, \ldots, 7\}$.
   Give all Maple commands (not the output!)

   (a) to show that p cannot be factored over $\mathbb{Z}_7$.

   (b) to compute a **numerical** factorization of p, using Maple's default precision;

   (c) to compute a **symbolic** factorization of p, using the proper extension of $\mathbb{Z}_7$.
   Give the factorization of p you obtain in this way.

/25

2. Consider `q := z*(x-y)/y`.

   (a) Give the Maple command (not its output!) to see the internal data structure of `q`.

   (b) Draw the directed acyclic graph which represents the internal data structure of `q`.

   (c) Do `subs(-1=1,q);` and explain the effect of this command.

/25

3. Explain why Maple often does not immediately evaluate an expression when it is typed in. Illustrate with an example where immediate evaluation is undesirable.

Give another good example where you as user have the choice to ask Maple either to evaluate the expression, or just to leave the expression as it is.

/10

4. Consider $p = 45x^{202} - 8x^{178} - 93x^{124} + 92x^{115} + 43x^{82} - 62x^{24}$.

   (a) How many arithmetical operations does it take to evaluate p?
   Give also the Maple command used to obtain this result.

   (b) Give the Maple command to convert p into an optimal form for evaluation.
   Do not give the output of the command.

   (c) How many operations are now needed to evaluate p in this optimal form?

/15

5. Consider $p = x^{10} - x^3 + 1$. Give all Maple commands (not the output!)

   (a) to compute all complex roots of p, with default 10-digit precision, assigning the result to `s`;

   (b) to create two lists, using `s` as input, called `real_parts` and `imag_parts`, which respectively contain the real and imaginary parts of the roots in `s`;

   (c) to merge the lists `real_parts` and `imag_parts` into one list of lists, called `points`, where every list in `points` consists of two elements `[r,i]`, with `r` the real and `i` the imaginary part of the same root.

/15

6. Give all Maple commands (not the output!)

   (a) to compute the coefficient of x^{123} in the Taylor expansion of e^x about $x = 0$;

   (b) to define a function, called `my_factorial`, using this Taylor expansion $\left(\text{i.e. : use } e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \right)$, which returns $n!$ for every n;

   (c) to return the number of digits in 123!. How many digits does 123! have?

/15

7. The Stirling numbers of the first kind $c(n,k)$ satisfy the recurrence

$$c(n,k) = (n-1)c(n-1,k) + c(n-1,k-1), \quad \text{for} \quad n \geq 1 \quad \text{and} \quad k \geq 1,$$

with the initial conditions that $c(n,k) = 0$ if $n \leq 0$ or $k \leq 0$, except $c(0,0) = 1$.

Write an *efficient* **recursive** procedure, call it `stirling1` to compute $c(n,k)$.

The n must be an index to `stirling1` while k is its argument, e.g.: for $n = 100$ and $k = 33$, `stirling1[100](33)` should return $c(100,33)$.

/20

8. (a) Explain the difference between the commands `diff` and `D` in Maple.

(b) Give an example where `diff` must be used instead of `D`.

(c) Give an example where `D` must be used instead of `diff`.

/15

9. Consider the system $\begin{cases} x^2 + y^2 - 2 = 0 \\ y - x^2 = 0 \\ z - x^3 = 0. \end{cases}$

(a) Give all Maple commands (not the output!) to convert this system into a triangular form.

(b) From the triangular form, explain how you can read off the number of complex solutions of this system. Also give the number of complex solutions.

(c) Give all Maple commands to compute all **rational** roots of this system. Also give the values for all rational roots you find.

/20

10. Give all **MATLAB** (*no Maple!*) commands to make a plot of the ellipse defined by $x = 2\cos(t)$ and $y = 3\sin(t)$, for $t \in [0, 2\pi]$.

/15

11. Write an m-file which returns the product of the function f applied to all elements in a vector, using the following prototype:

```
function p = product(f,x)
%
%  returns f(x(1))*f(x(2))*f(x(3)) ...,
%  the product of f evaluated at all elements of the row vector x.
%
```

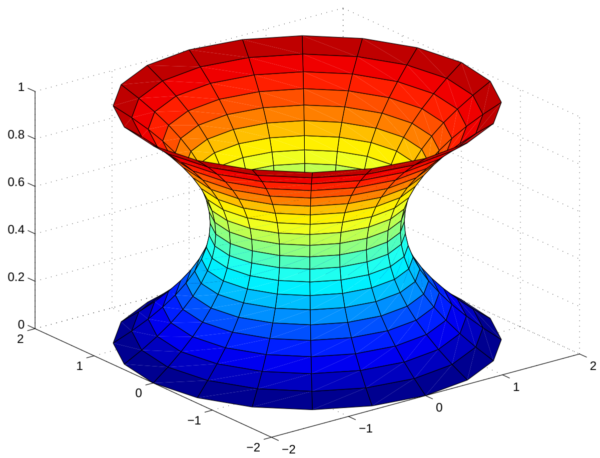
Give all MATLAB commands to use `product` to compute $\cos(1) \times \cos(2) \times \cdots \times \cos(10)$, i.e. `f = cos` and `x` is `[1 2 3 4 5 6 7 8 9 10]`.

Give also the value MATLAB returns after executing this call to `product`.

/15

12. Consider a surface symmetric around the vertical z-axis, where the distance to the z-axis is defined by $1 + z^2$, i.e.: the surface is closest to the z-axis when $z = 0$.

Give all MATLAB commands to produce the following hyperboloid of one sheet:



/10