

**Math 215. Midterm 2**  
Spring 2008, A.Libgober

*If you use a theorem or a general fact proved in the text give exact statement of the fact your answer is based on*

1. Determine if the function  $f : \mathbf{R} \rightarrow \mathbf{R}$  given by  $f(x) = x^2 - 4x$  is injective, surjective or bijective. If the answer is “no” to either of the questions find subsets of the domain and codomain so that for the function given by the same formula but with changed domain and codomain the answer is “yes”.

2. Let  $X = \{a, b, c\}$  and  $Y = \{c, d, e\}$ . Let  $n$  be the cardinality of  $X \times Y$ . What is  $n$ ? Describe a bijection  $\mathbf{N}_n \rightarrow X \times Y$ .

3. Find the coefficient of  $a^{85}b^{15}$  in  $(a + b)^{100}$ . This coefficient is equal to the number of certain type of subsets of a set with 100 elements. What kind of subsets this coefficient counts?

4. Find the rational number equal to the recurrent infinite decimal 23.564564564....

5. Let  $|X| = 100$  and  $|Y| = 2$ . What is the cardinality of the set of functions from  $X$  to  $Y$ . How many subsets  $X$  has?. Does exist a bijection between these two sets. If the answer is yes, describe a bijection.

6. Give definition of a well ordered set. Is the set of even integers a well ordered set? Is  $\mathbf{R}_{\geq 0}$  (non negative real numbers) a well ordered set?