MTHT 435. Fall 2007
Abstract Algebra
Final exam
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1. Prove that $\mathbf{Q}[x]/ < x^2 - 7 >$ is ring-isomorphic to $\mathbf{Q}(\sqrt{7})$.

2. Let $f(x) = 5x^4 + 3x^3 + 1$ and $g(x) = 3x^2 + 2x + 1$ be polynomials in $\mathbf{Z}_7[x]$. Determine the quotient and remainder upon dividing $f(x)$ by $g(x)$.

3. Write $x^3 - 1$ as a product of irreducible polynomials over $\mathbf{Z}_7$.

4. Find the degree of extension $\mathbf{Q}(\sqrt{2} + \sqrt{5})/\mathbf{Q}$.

5 Write $(2 + 4\sqrt{3})^{-1}$ in the form $a + b\sqrt{3}$ $(a, b \in \mathbf{Q}.)$

6. Find the degree and a basis of $\mathbf{Q}(\sqrt{2} + \sqrt{3})$ over $\mathbf{Q}(\sqrt{6})$ (resp. over $\mathbf{Q}$).

7.a) Calculate the order of alternating group A_5.
b)Find the order of the permutation $(125)(4376)$.

8. Find the degree of the extension $\mathbf{Q}(cos40^\circ)/\mathbf{Q}$ and $\mathbf{Q}(cos15^\circ)/\mathbf{Q}$. Show that angle 40° is not constructible but the angle 75° is.