Math 516 Abstract Algebra

A.Libgober, Fall 2003

1.Describe all groups of order 93.

2.Find prime factorization of 5 in the ring of Gaussian integers and present an argument which shows that each factor is indeed a prime.

3.Find all monic irreducible polynomials in $\mathbf{F}_3[x]$ having degree ≤ 3.

4.Let e, f be a basis of a two-dimensional vector space. Show that $e \otimes f + f \otimes e$ cannot be written as a decomposable tensor $v \otimes w$.

5.Consider the commutative diagram:

$$\begin{array}{ccccccc} A & \to & B & \to & C & \to & D \\ \alpha \downarrow & & \beta \downarrow & & \gamma \downarrow & & \delta \downarrow \\ A' & \to & B' & \to & C' & \to & D' \end{array}$$

in which the rows are exact. Show that if α is surjective and β and δ are injective then γ is injective.