Math 517. Spring 2009
Abstract Algebra. Final exam
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1. Prove that two 3×3 matrices are similar if and only if they have the same characteristic and minimal polynomials. Give an explicit counterexample to this assertion for 4×4 matrices.

2. Describe the Galois group of $x^4 - 7$ over $\mathbf{Q}$ as a subgroup of permutation group of the roots.

3. Let $V = \mathcal{Z}(xy - z) \subset \mathbf{A}^3$. Show that V is isomorphic to $\mathbf{A}^2$.

4. Show that the quotient of $SL_2(\mathbf{F}_3)$ by its center is the alternating group A_4 and use it to prove that $H^2(A_4, \mathbf{Z}_2) \neq 0$.

5. Prove that a finitely generated abelian group A is free if and only if $Ext^1(A, \mathbf{Z}) = 0$.