Math 517. Spring 2004
Abstract Algebra. Midterm 1
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1. Show that $x^5-1=(x-1)(x^2-4x+1)(x^2+5x+1)$ in $\mathbf{F}_{19}[x]$. Use this to determine up to similarity al 2×2 matrices with entries in $\mathbf{F}_{19}$ having multiplicative order 5. Find the number conjugacy classes of matrices of order 5 over $\mathbf{C}$.

2. Determine the Jordan canonical form for the matrix:

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3. Show that is N is a $n\times n$ nilpotent matrix then $N^n=0$.

4. Show that A is an endomorphism of a finite dimensional vector space then:

$$exp(\sum_{r=1}^{\infty} Tr(A^r)\frac{t^r}{r}) = det(1-At)^{-1}$$

5. Let $M=\mathbf{Z}^n$ and $N=\{(b_1,...,b_n)\in M|\sum_i b_i = 0\text{mod n}\}$. Find a basis $e_1,...,e_n$ in M and integers $a_1,...,a_n$ such that $a_1e_1,...,a_ne_n$ is a $\mathbf{Z}$-basis in N.

6. Find the degree of $\mathbf{Q}(\sqrt{2}+\sqrt{3})$ over $\mathbf{Q}$ and the minimal polynomial of $\sqrt{2}+\sqrt{3}$.

7 Find the degree of the splitting field of x^4+2 over $\mathbf{Q}$.

8 Show that non zero elements of a field with 9 elements form a cyclic group.