Math 554. Fall 2007
Complex Manifolds, Final exam
A.Libgober

1. Let $X = \mathbf{P}^1 \times \mathbf{P}^1$ Let x_0, x_1 and (y_0, y_1) be homogeneous coordinates on the first and second factors respectively and let p be the bi-homogeneous polynomial

$$x_0^d(y_0^l + y_1^l) + x_1^d(y_0^l - y_1^l) \qquad (*)$$

a) Using the standard cover of $\mathbf{P}^1$ (by two open sets given by non vanishing of one of the homogeneous coordinate) construct a cover of X by four open sets each biholomorphic to $\mathbf{C}^2$.

b)Show that equation (*) defines the subset C of X and find defining equation in each of the charts you construct in a). Explain why a polynomial in $(x_0.x_1, y_0, y_1)$ which is not bi-homogeneous does not have a well defined zero set.

c)Show that C is a sub-manifold of X.

d) Consider the projection of C on $\mathbf{P}^1$ which is one of the factors of X. Find the number of preimages each point of $\mathbf{P}^1$ has. Use additivity of euler characteristic to determine the genus of C.

2. Construct a complex manifold homeomorphic to $S^1 \times S^5$ (here S^n denotes the n=dimensional sphere).

3. Find the degree of the image of Segre embedding $\mathbf{P}^2 \times \mathbf{P}^{\mathbf{2}} \to \mathbf{P}^N$. Determine the integer N.

4, Let E be a holomorphic vector bundle on a complex manifold X. Let U be an open set over which holomorphic bundle E is trivial. Describe the action of $\bar{\partial}_E$ on a C^∞ section of $E|_U$ and explain why the result is independent of holomorphic change of trivialization.

5. Describe Chern form of the bundle $\mathcal{O}_{\mathbf{P}^1}(1)$ and Chern form of the bundle given the Segre embedding of X from problem 1.