3 May 2001

1.

2.

3.

4. $\nabla f(1,-2,1) = \langle -6, 2, -8 \rangle$ and $\mathbf{u} = \frac{1}{\sqrt{3}} \langle 1, 1, -1 \rangle$ so that $\mathbb{D}_{\mathbf{u}} f = \frac{4}{\sqrt{3}}$.

5. (a) Saddle at $(0, 1)$. Here, $D = -4$.

   (b) Maxima at $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ and $\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$, min at $(0, -1)$.

6.

$$\int_0^1 \int_0^{\sqrt{y}} x \sin y^2 \; dx \; dy - \frac{1}{4} (\cos 1 - 1) .$$

7. 12π.

8.

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_{-1}^{1} \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) = \int_{-1}^{1} \left\langle 2t^3, t^3, 2t^2 \right\rangle \cdot \langle 1, 2, 2t \rangle = \int_{-1}^{1} 8t^3 = 0.$$

9. (a) $\frac{\partial P}{\partial y} = x^2 e^{xy} \neq y^2 e^{xy} = \frac{\partial Q}{x}$

   (b) $f(x, y) = e^{xy}$

   (c) 0

10.

$$\int_0^2 \int_1^3 (2xe^y - x) \; dx \; dy = 8e^2 - 16.$$