# Quiz 1

1. (Exercise 24, §13.5) Find an equation for the plane $\Pi$ through the point $P = (4, 0, -3)$ with normal vector $\mathbf{n} = \mathbf{j} + 2\mathbf{k}$.

   **Answer:** Let $\mathbf{r}_0 = \langle 4, 0, -3\rangle$ and let $(x, y, z)$ be in $\Pi$. $(x, y, z)$ corresponds with the vector $\mathbf{r} = \langle x, y, z\rangle$ and we would therefore have that $\mathbf{r} - \mathbf{r}_0$ is "contained" in $\Pi$. Then $\mathbf{r} - \mathbf{r}_0$ would be orthogonal to $\mathbf{n}$ so that

$$\begin{aligned} 0 &= (\mathbf{r} - \mathbf{r}_0) \cdot \mathbf{n} \\ &= \big( \langle x, y, z\rangle - \langle 4, 0, -3\rangle \big) \cdot \langle 0, 1, 2\rangle \\ &= \langle x - 4, y, z + 3\rangle \cdot \langle 0, 1, 2\rangle \\ &= y + 2z + 6. \end{aligned}$$

2. (Exercise 15, §13.5) Find symmetric equations for the line $\ell$ that passes through the point $(0, 2, -1)$ and is parallel to the line given by

$$\begin{cases} x = 1 + 2t \\ y = 3t \\ z = 5 - 7t \end{cases} \tag{1}$$

   Find the points in which $\ell$ interects the $xy$-, $xz$-, and $yz$-planes.

   **Answer:** The line given in (1) has vector equation

$$\mathbf{r}_0 + t\mathbf{v}$$

   where $\mathbf{r}_0 = \langle 1, 0, 5\rangle$ and $\mathbf{v} = \langle 2, 3, -7\rangle$. (Verify this!) Now since $\ell$ is parallel to this line, $\ell$ must have the same direction part $\mathbf{v}$. We can then take $\ell$ to be given by the vector equations

$$\mathbf{r}_1 + t\mathbf{v}$$

   where $\mathbf{r}_1 = \langle 0, 2, -1\rangle$. In coordinates, $\ell$ is given by

$$\langle 2t, 2 + 3t, -1 - 7t\rangle \qquad \text{or} \qquad \begin{cases} x = 2t \\ y = 2 + 3t \\ z = -1 - 7t \end{cases} \tag{2}$$

   so that the symmetric equations for $\ell$ are

$$\frac{x}{2} = \frac{y - 2}{3} = \frac{z + 1}{-7}.$$

   $\ell$ intersects the $xy$-plane at the point where $z = 0$. Solving the third equation in (2) for $t$, we have

$$0 = z = -1 - 7t \qquad \Longrightarrow \qquad t = \frac{-1}{7}.$$

   Substituting the value $t = \frac{-1}{7}$ into the other equations in (2), we have $x = \frac{-2}{7}$ and $y = \frac{11}{7}$. This means that $\ell$ intersects the $xy$-plane at the point $\left(\frac{-2}{7}, \frac{11}{7}, 0\right)$. Similarly, $\ell$ intersects the $yz$-plane at $\left(\frac{-4}{3}, 0, \frac{11}{3}\right)$ and intersects the $yz$-plane at $(0, 2, -1)$.