Quiz 1

1. Find two *unit* vectors which are orthogonal to *both* $\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} + \mathbf{k}$.

   Answer: The vectors

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$$

   and

$$\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = -\mathbf{i} - \mathbf{j} + 2\mathbf{k}$$

   are both orthogonal to both $\mathbf{u}$ and $\mathbf{v}$. Normalizing, we have that

$$\frac{\mathbf{i} + \mathbf{j} - 2\mathbf{k}}{|\mathbf{i} + \mathbf{j} - 2\mathbf{k}|} = \frac{\mathbf{i} + \mathbf{j} - 2\mathbf{k}}{\sqrt{6}}$$

   and

$$\frac{-\mathbf{i} - \mathbf{j} + 2\mathbf{k}}{|-\mathbf{i} - \mathbf{j} + 2\mathbf{k}|} = \frac{-\mathbf{i} - \mathbf{j} + 2\mathbf{k}}{\sqrt{6}}$$

   are *unit* vectors which are orthogonal to *both* $\mathbf{u}$ and $\mathbf{v}$.

2. Recall that the *angle* between two intersecting planes is the angle between the vectors which are normal to the planes. Two intersecting planes are *perpendicular* if the angle between them is is $\pi/2$. Determine whether the planes π_1 given by $x + y + z = 1$ and π_2 given by $x - y + z = 1$ are parallel, perpendicular, or neither. If neither, find the angle between them.

   Answer: The vector $\mathbf{n}_1 = \langle 1, 1, 1 \rangle$ is normal to π_1 and the vector $\mathbf{n}_2 = \langle 1, -1, 1 \rangle$ is normal to π_2. If θ is the angle between $\mathbf{n}_1$ and $\mathbf{n}_2$, then

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|\,|\mathbf{v}|} = \frac{1}{\sqrt{3}\sqrt{3}} = \frac{1}{3}$$

3. Find *symmetric equations* for the line ℓ that passes through the points $(6, 1, -3)$ and $(2, 4, 5)$.

   Answer: The vector

$$\mathbf{v} = \langle 6 - 2, 1 - 4, -3 - 5 \rangle = \langle 4, -3, -8 \rangle$$

   goes from $(6, 1, -3)$ to $(2, 4, 5)$, so ℓ should have direction vector ℓ. Then ℓ is given either by

$$\frac{x - 6}{4} = \frac{y - 1}{-3} = \frac{z + 3}{-8}$$

   or

$$\frac{x - 2}{4} = \frac{y - 4}{-3} = \frac{z - 5}{-8}$$